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# Issues, Concepts, and Standard Techniques in Assessing Accuracy of Coordinate Measuring Machines

Dennis A. Swyt

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[1] At Boulder, CO 80303.
[2] Some elements at Boulder, CO 80303.

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# *Issues, Concepts, and Standard Techniques in Assessing Accuracy of Coordinate Measuring Machines*

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## Preface

Prepared for use by the Department of Energy's Oak Ridge Facilities, this report deals with a variety of issues, concepts, and standard techniques in assessing the accuracy of measurements of modern laser-based computer-controlled coordinate measuring machines. The report brings together the substance of three closely related works by the author dealing successively with the issues, concepts, and techniques.

The first work and chapter of this report, after defining dimensions and tolerances of manufactured parts and showing a major trend to tightening of tolerances, outlines briefly some of the issues in assessing conformity of dimensions of manufactured parts to design tolerances in terms of five topical areas [1]. These five topics are: 1) mathematicization of geometric dimensions and tolerances; 2) evaluation of the computational aspects of computerized measuring systems; 3) the evaluation of the physical measurement aspect of measuring machines; 4) assignment of uncertainty to measurement results; and 5) estimation of manufacturing deviations relative to tolerances in the presence of measurement uncertainty.

The second work and chapter of this report presents a new scheme for the analysis of the dimensional capabilities of measuring machines and machine tools aimed at the realization of higher degrees of precision in the manufacture of dimensioned parts [2]. Successive sections deal with the range and types of dimensional measurements, their relation to the basic unit of measure, the means for making such measurements, and the errors associated with them. Application of these principles is illustrated by a detailed analysis of the use of a laser-based coordinate measuring machine for the measurement of a physical part. Also included is an indication of how the identical analysis may be applied to a machine tool which shapes such parts.

The third work and chapter of this report compares U.S., German, British, and Japanese standards for evaluation of coordinate measuring machines [3]. It also shows that using a variety of special artifacts the U.S. standard alone specifies separate tests for each of the four modes of length measurements (displacement, position, distance, and extension) and that, depending on the number and orientations of faces probed, a bi-directional step gage of the type suggested in the other standards can be used for tests of each of these.

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**Abstract**

This report deals with a variety of issues, concepts and standard techniques in assessing the accuracy of measurements of modern laser-based computer-controlled coordinate measuring machines. It outlines technical issues in assessing conformity of dimensions of manufactured parts to design tolerances with such CMM's, uses a new system of length-dimensional types (displacement, position, distance, and extension) as the basis for an error-budget analysis of CMM measurement capabilities and, based on that system, intercompares United States, German, British, and Japanese CMM performance standards, showing that the U.S. standard provides separate test of a CMM's capability to measure of each length-dimensional type and the part features associated with them.


**Key Words:** accuracy; CMM's; coordinate measuring machines; coordinate metrology; displacement; distance; extension; length; manufacturing tolerances; measurement; performance evaluation; position; standards; testing

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# 1    Issues in Assessment of Conformity of Dimensions of Manufactured Parts to Design Tolerances

## 1.1  Introduction

In the manufacture of discrete parts, from automobile engine blocks to quantum-well electronic devices, a principal determinant of quality is conformity of geometrical dimensions of the part as manufactured to those of the part as designed. The degree of this conformity is determined by the relationship of measured deviations of geometrical dimensions to specified tolerances.

Today the historical system of manufacture — which involves relatively low precision, hard gaging of part elements, graphic representations of tolerances, and a willingness to accept high scrap rates — is being supplanted by a new system of manufacture — which involves high-precision manufacture, computer-based coordinate-metrology, mathematical representations of tolerances, and demand for making every part right the first time. This new system has raised major issues for research and standardization across the spectrum of design specification, precision manufacture, and measurement-based quality assessment.

After defining dimensions and tolerances of manufactured parts and showing a major trend to tightening of tolerances, this chapter outlines briefly some of the issues in assessing conformity of dimensions of manufactured parts to design tolerances in terms of five topical areas: 1) mathematicization of geometric dimensions and tolerances; 2) evaluation of the computational aspects of computerized measuring systems; 3) the evaluation of the physical measurement aspect of measuring machines; 4) assignment of uncertainty to measurement results; and 5) estimation of manufacturing deviations relative to tolerances in the presence of measurement uncertainty.
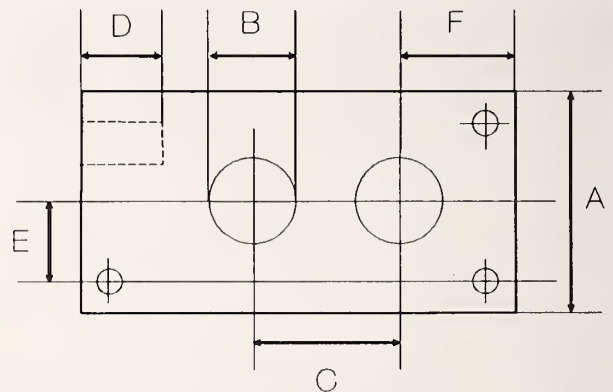


**Figure 1.    An engineering drawing representation of principal geometrical dimensions of manufactured discrete part.**

## 1.2  Dimensions of Manufactured Discrete Parts

Whether aircraft, automobiles, computers, or integrated circuits, discrete-part products are able to function and to function properly because the components of these products have specific geometrical shapes, sizes, and related dimensions.

Figure 1 is an engineering drawing representation of an automobile engine which illustrates some of the principal types of geometrical dimensions of discrete parts.  These types of geometric features include:  A, representing a length or width of the engine block; B, representing the diameter of a cylinder bore; C, representing the distance between the centers (axes) of cylinder holes; D, representing the depth of a blind hole; E, representing the location of the center (axis) of a hole relative to a plane; and F, representing the (true) position of a feature relative to the origin of a part coordinate system defined by, for example, reference holes in the oil pan plane.

## 1.3  Tolerances on the Dimensions of Discrete Parts

As defined by U.S. ANSI Standard Y14.5 on "Geometric Dimensioning and Tolerancing," a tolerance is a number given in an engineering drawing which specifies the total amount by which a dimension is permitted to vary [4]. Represented here by $\pm T$, a tolerance defines a range of acceptability of the magnitude of the difference between the dimension of a part-as-made $(D_M)$ and the dimension of the part-as-designed $(D_D)$, that is, $| D_M - D_D | \leq T$.

Figure 2 shows tolerance-versus-dimension for selected examples of the wide variety of outputs of the discrete-parts manufacturing industry, spanning a range of dimensions from 1 m to 10 nm and a range of tolerances from 1 millimeter to 0.5 nanometers [5].



Tolerances vs Dimensions of
Current State-of-Art Mfd Parts

Figure 2.  Log-log plot of tolerance versus dimension for discrete-part products in normal, precision, and ultraprecision regimes.

At the large-dimension end of the precision-manufacturing spectrum is the state-of-the-art in automobile manufacture where a 1 mm tolerance on the fit of 1 m size passenger-car door assemblies is achieved only by the Japanese.  Next, also in state-of-the-art automobile manufacturing, is the 7-8 μm

tolerance on metal-matrix composite pistons for a new-generation engine under development by a leading Japanese car company. In the intermediate-dimension range is the 0.25 µm accuracy of a slotting/slicing machine for use in the fabrication of the new-generation thin-film magnetic read/write heads for computer memory systems. Toward the lower dimensions are the 0.025 µm (25 nm) tolerances on 125 µm diameter optical fibers for telecommunications systems and the 0.010 µm (10 nm) tolerances representing current demands in electron-beam micropatterning equipment in Japan. Finally at the smallest-dimension end of the precision-manufacturing spectrum shown are 0.0005 µm (0.5 nm) variations on the 10 nm size dimensions of devices such as quantum-well lasers as well as quantum-wire and quantum-dot devices under development.

There is an ongoing trend toward dramatically tighter dimensional tolerances in manufactured goods — first reported to the international production community in the early 1980's — which corresponds to a roughly factor-of-three decrease in the size of tolerances every 10 years [6]. According to the projection of this trend, between 1980 and 2000 state-of-the-art tolerances will decrease: from 7.5 µm to 1 µm in the most-demanding "normal" machine-tool-based production; from 0.075 µm to 0.01 µm in the most-demanding "precision," e.g., diamond turning machine production; and from 0.005 µm (5 nm) to < 0.001 µm (< 1 nm) in the most-demanding atom-, electron-, or x-ray "machining"-based production. It is the trend to such tightened tolerances, combined with the move in many inspection applications towards use of computerized coordinate measuring machines, that has given rise to major current metrological problems in measurement-based quality assessment of precision-manufactured discrete-part products.

## 1.4  Problems in Assessment of Conformity to Tolerances

Shown schematically in Table 1 the principal aspects of the overall problem of determining with a known degree of confidence whether the dimensions of a manufactured part conform within specified tolerances in an engineering design. As indicated in Table 1, with each of the functional tasks involved in design, fabrication, point-sample-based measurement, and part evaluation, there is an associated variability which in the best situation is appropriately quantified.

According to this scheme, a bottom-line evaluation of a part consists of the ability to state with confidence **C** that a part's material deviations **D**, known with measurement uncertainty **U**, are within design-specified tolerances **T**, when the deviations have been determined by means of a point-sample measuring machine which provides coordinate positions with errors **X**, of a number of points distributed at various locations on the surface of the part, when those points are used in conjunction with a software implementation of a fitting algorithm with associated variability **Y**.

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**Table 1.** Principal tasks and their associated variabilities in comparing point-sample measurements of manufacturing deviations with design tolerances

| Function | Tasks | Variability | Symbol |
|----------|-------|-------------|--------|
| Design | Choose Ideal Form | | |
| | Specify Dimensions | Design Tolerances | T |
| Fabrication | Choose Process | | |
| | Shape Material | Mfg Deviations | D |
| Data-Taking | Choose Number/Location of Points | | |
| | Probe Surface at Those Points | Coordinate Errors | X |
| Computation | Choose Fitting Algorithm | | |
| | Compute Substitute Geometry | Fitting Variation | Y |
| Evaluation | Calculate Manufacturing Deviations | Uncertainty | U |
| | Decide If Within Tolerances | Confidence | C |

## 1.4.1 Design: Mathematicization of Geometric Dimensions and Tolerances

In the part design phase of Table 1, the basic tasks are to choose an ideal geometric form for the part and to specify its dimensions and tolerances. Current practice and standards on geometric dimensioning and tolerancing, represented by ANSI Y14.5, deal with graphical, engineering-drawing representation of geometry, dimensions, and tolerances. The first major issue associated with the assessment of part manufacturing deviations relative to tolerances by means of computerized point-sample coordinate measuring systems arises with the tolerances themselves. Absent are the formal representations required for unambiguous interpretation and computation [7]. The required "mathematicization" of dimensioning and tolerancing standards is an area of active research underlying new standards [8]. Problems include, for example: achievement of context-free definitions of concepts such as *size*, which do not involve embedded notions such as *diametrically-opposed points* or fitted *cross sections* [9]; and means for establishing simultaneously formal and common-sense bases for concepts such as the various types of *datum* to which part features are referred [10].

## 1.4.2 Data Taking: CMM Measurements

Given that a manufacturing process has been chosen which is likely to produce minimum deviations from design values and the part fabricated, in the next step — that of data taking — the main tasks are to choose the number and distribution of points about the part surface at which coordinates are to be measured and to actually probe the part. Two issues arise.

The first issue is appropriate selection of the number and location of sample points on the surface of the part being measured. Approaches include, for example, equidistant, equiangular, and stratified. Some standards provide guidelines for sampling [11]. Active research on the design of sampling strategies as a function, for example, of part geometry, process-specific part form errors, and systematic and random probing errors is being carried out as potential input to U.S. standardization on dimensional measurement inspection techniques [12].

The second issue is taking into account the residual measurement errors associated with each individual probed point. Recent work has led to standard techniques for characterizing these specific functional aspects of coordinate measuring machines [13]. These residual errors, which are vectors of varying direction and magnitude specific to each probed point, are complex functions of the coordinate measuring machine geometry, location in its working volume, type of probe, and direction of approach of the probe to the part surface [14]. Ongoing research includes that on the characterization of the errors of CMM probes which can have systematic, direction-dependent



**Figure 3.** Lobing in response of CMM probe which gives rise to directional errors in coordinates of points on part being measured (full scale of top and bottom plots are 8 μm and 14 μm respectively).

errors such as the lobing shown in figure 3 [15], with an ideal probe having a circular, i.e., non-direction-dependent error pattern of zero radius. As can be inferred by comparing figure 1 with figure 3, the cumulative effect of such lobed, direction-dependent errors on the apparent dimension of a part being measured depends systematically upon which of the six different types of geometric features is being probed, the number of points probed, and the specific directions of approach to the part relative to the probe's lobing pattern.

### 1.4.3 Computation:  Fit Criteria and Algorithm Testing

In the computation phase of the scheme of table 1, the basic tasks are to choose an algorithm for achieving a best fit of the set of points measured on the surface of the part and to compute the best-fit substitute geometric feature.  Conventional fitting criteria include, for example, least squares, mini-max, maximum inscribed and minimum circumscribed.  The issue here is that computed dimensions of the substitute feature, including its size, location and orientation, depend on the substitute geometric element selected, the best-fit criteria applied, and the software implementation of the fitting algorithm — with no established theory of tolerances to rationalize the choice.  The area is one of active research with implementations of algorithms based on specific criteria, such as a minimum deviation zone, being evaluated against simulated measurement points for specific substitute geometries such as cylinders [16].  Also under development are standardized tests for the evaluation of the ability of various substitute-geometry fitting algorithms to detect particular types of form errors [17].

### 1.4.4  Evaluation:  Measurement Uncertainty and Confidence

Finally, in the evaluation phase of the assessment of part manufacturing deviations relative to tolerances by means of computerized point-sample coordinate measuring systems as shown in table 1, there are the tasks of actually comparing the substitute geometry with the design geometry to determine whether the part's manufacturing deviations are within specified design tolerances, given the overall uncertainty in measurement.  The conventional approach in dealing with assessing manufacturing deviations relative to tolerances in the presence of measurement uncertainty is to require that the accuracy of measurements be some specified fraction of tolerances, typically one-fourth (as in the case of the U.S. Department of Defense prescription) or one-tenth (as in the case of the widely applied "gage maker's rule").  However, there is a major issue here in ascertaining conformance of manufactured parts to design tolerances in that one does not know how to calculate the confidence factors of the uncertainties in the overall measurement based on coordinate and sampling errors [5].

The main issue is compounded by secondary issues.  First is the absence of a statistically sound and  widely accepted measure of measurement uncertainty.  At present there is a move away from the long-established system of characterizing uncertainty in terms of a random component (in terms of one-, two-, or three-standard deviations) and a systematic component (in terms of an estimated residual after model-based systematics are removed) long used in the United States [18] to a quadrature-addition, one-sigma representation being pursued by the international metrology organizations [19], with the two approaches seen to have more or less utility depending on the application [20].  In any case, the question of calculating confidence in measurement uncertainty is still open.

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## 1.5 Conclusion

Achievement of quality in design of discrete parts requires specification of tolerances on manufacturing deviations which can be achieved to an expected degree of confidence. With the press of tightening tolerances and the use of point-sampling coordinate measuring machines, achievement of a known confidence is a problem. Solution of that problem requires resolution of a wide range of issues involving mathematicization of dimensioning and tolerancing standards, development of point sampling strategies, analysis of fitting algorithms, testing of software implementations, and development of error propagation and confidence estimation techniques. While aspects of the overall problem are being addressed within the research and standards communities, much work remains to be done.

## 2    New Concepts of Precision Dimensional Measurement for Modern Manufacturing

### 2.1  Introduction

A hallmark of modern products is the high precision of the dimensions of their functional parts — high, that is, compared to that of their less-modern contemporaries. Since the Industrial Revolution, such has been the case. Modern products of the last century included certain factory-produced small arms, the precision of whose parts allowed them to be interchangeably assembled in the then-new system of mass production. Modern products of this century are video cassette recorders, the precision of whose parts allows read-write heads to be aerodynamically flown over the recording medium at microscopic altitudes.

Since it is modern products which sell best, for U.S. manufacturing firms seeking to compete better against evermore sophisticated foreign producers, the ability to realize precision higher than that of their competitors is essential. Precision, however, is relative not only to the capabilities of the competition, but also to the size of the functional parts in question. Comparable degrees of high precision are a common end for a wide range of manufacturers, from producers of the very large, such as commercial aircraft, to producers of the very small, such as nanoelectronic devices.

This chapter presents a new scheme for the analysis of the dimensional capabilities of measuring machines and machine tools aimed at the realization of higher degrees of precision in the manufacture of dimensioned parts. Successive sections deal with the range and types of dimensional measurements, their relation to the basic unit of measure, the means for making such measurements, and the errors associated with them. Application of these principles is illustrated by a detailed analysis of the use of a laser-based coordinate measuring machine for the measurement of a physical part. Also included is an indication of how the identical analysis may be applied to a machine tool which shapes such parts.

## 2.2 Scope of Dimensional Measurements

Manufacture of today's most modern products requires the ability to carry out precise dimensional measurements over a wide scope, one which includes subtly different types and astonishingly different scales. Dimensional measurements of all types and scales, however, are expressed in terms of the basic unit of length.

### 2.2.1 The Range of Dimensional Measurements

The range over which precision dimensional measurements for the characterization of manufactured goods are made today spans nearly 12 orders of magnitude, segmented into relatively well recognized regimes.

#### 2.2.1.1 The Macro-Scale

On the large-size end of the range are products which are not so much manufactured as constructed, that is, large-scale products often assembled on site rather than manufactured on line. Macro-scale products such as ships, aircraft, and spacecraft can have longest dimensions of the order of 100 meters.

#### 2.2.1.2 The Mid-Scale

In the mid-size of the range are products the parts of which are manufactured and assembled in the familiar factory environment, including a host of commonplace products from automobiles to machine screws. Mid-scale products have characteristic dimensions of the order of multiple- to fractional meters.

#### 2.2.1.3 The Micro-Scale

On the conventional small-size end of the range are products which are manufactured within special machines such as optical, electron-beam, and x-ray lithography systems, including less familiar products such as ultra-large-scale-integration (ULSI) microelectronic devices. Micro-scale products have characteristic dimensions often of less than a micrometer.

#### 2.2.1.4 The Nano-Scale

Just emerging now is technology for the manufacture of products of the "nano-scale," including nanoelectronic (as opposed to microelectronic) transistor-type devices 100 times smaller than those in present commercial production and mechanical devices such as ultra-miniature pumps capable of implantation into blood vessels and automatically metering out drugs. Nano-scale structures and devices have features composed of small numbers of atoms

and molecules and have characteristic dimensions of the order of nanometers ($10^{-9}$ m, or 40 billionths of an inch). Achievement of high precision in dimensional measurements at the nanometer scale demands utmost attention to physical-scientific, engineering, and metrological principles [21].

## 2.2.2 Types of Dimensional Measurements

In the characterization of today's most modern manufactured products, precision dimensional measurements are carried out not only over a wide range of dimensions but also over a range of types, each of which, while intimately related to the others, has a uniquely distinct role in measurement.

### 2.2.2.1 Position

**a. *Geometrical Concept of Position*:** In geometry, position is the location of a point relative to either: another point, such as the origin of a one-dimensional coordinate system; a line, such as an axis of a two-dimensional coordinate system; or a plane, such as the reference surface in a three-dimensional coordinate system. For a three-dimensional cartesian system, the formally explicit expression for geometric position is a vector quantity:

$$p = p\,(xx, yy, zz) \tag{1}$$

where $x$, $y$, and $z$ represent the coordinates along unit vectors $x$, $y$, $z$ relative to the (suppressed null-vector) origin of the coordinate system.

**b. *Physical Concept of Position*:** (In physics, position is the location in space of a physical body relative to a point of observation.) In the rigid-body approximation, where the configuration of that body is fully specified by six generalized coordinates (three linear and three rotational) corresponding to its six degrees of freedom, position is given by the three linear coordinates. Note that since both a rigid body and a reference frame have the same six degrees of freedom, locating a body relative to a reference frame is fully equivalent to locating one rigid body relative to another. In a three-dimensional cartesian system, the formally explicit expression for physical position is a vector quantity:

$$P = P\,(XX, YY, ZZ) \tag{2}$$

where $X$, $Y$, and $Z$ represent the coordinates of the location of one physical body relative to the location of another physical body which acts as the reference frame of the coordinate system.

**c. *Measurement Concept of Position*:** In measurements, position is the assigned numerical value of the length of path between the single point which describes the location of a

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physical body and the origin, axis, or plane of the one-, two-, or three-dimensional coordinate system expressed in terms of the standard unit of measure. In a three-dimensional cartesian system, the formally explicit expression for measurement position is a vector quantity:

$$P = P ( XX, YY, ZZ) \tag{3}$$

where X, Y, and Z, expressed in meters, represent the coordinates of the location of one physical body relative to the location of another physical body which acts as the reference frame of the coordinate system. Measurements of position are made, for example, in precision surveying of geodetic coordinates relative to control points and in measurements of the locations of points on objects by means of coordinate measuring machines.

## 2.2.2.2 Displacement

a. *Geometrical Concept of Displacement*: In geometry, there is nothing fully equivalent to displacement. However, the physical notion of displacement does involve the geometrical notion of translation, where translation is the transformation of a point at one position into a point at a different position by means of the operation:

$$t = p_f - p_o \tag{4}$$

where $t$ is the vector of translation and $p_o$ and $p_f$ are the initial and final position vectors of the point.

b. *Physical Concept of Displacement*: In physics, displacement is the change in location in space of a physical body with time, that is, its movement from one location to another, and is described by:

$$T = P_f (t_1) - P_o (t_2), \tag{5}$$

where $T$ is the translation representing the change in position with time of a single body from position $P_o(t_1)$ at time one to position $P_f(t_2)$ at time two.

c. *Measurement Concept of Displacement*: In measurements, displacement is the assigned numerical value of the length of path between the location of a physical body at an initial time and its location at a final time expressed in terms of the standard unit of measure. In a three-dimensional cartesian system, the formally explicit expression for measured displacement D, expressed in meters, would be a vector quantity:

$$D = P_f (t_1) - P_o (t_2), \tag{6}$$

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where the difference between $\mathbf{P}_o(t_1)$ and $\mathbf{P}_f(t_2)$ represents the change in location, between times one and two, of one physical body relative to the location of another physical body (which acts as the reference frame of the coordinate system). Measurements of displacement are made, for example, in the characterization of the travel of a stage by means of a laser interferometer, where the stationary parts of the interferometer comprise the body which defines the reference frame.

## 2.2.2.3  Distance

**a.** *Geometrical Concept of Distance:*  In geometry, distance is the separation between two points.  In a three-dimensional cartesian system, the formally explicit expression for geometric distance would be a scalar quantity:

$$d = \mid p_1 (x_1 x,\ y_1 y,\ z_1 z) - p_2 (x_2 x,\ y_2 y,\ z_2 z) \mid,\tag{7}$$

where x, y, and z represent the values of measured coordinates relative to the usually suppressed null-vector origin of the coordinate system.

**b.** *Physical Concept of Distance:*  In physics, distance is the separation in space of the locations of two objects.  In a three-dimensional cartesian system, the formally explicit expression for physical distance would be a scalar quantity:

$$d = \mid P_1(X_1 X,\ Y_1 Y,\ Z_1 Z) - P_2(X_2 X,\ Y_2 Y,\ Z_2 Z) \mid,\tag{8}$$

where $d$ is the distance between two bodies located at positions $P_1$ and $P_2$ respectively.

**c.** *Measurement Concept of Distance*:  In measurements, distance is the assigned numerical value of the length of path between the location of one physical body and the location of another, expressed in the standard unit of measure.  A formally explicit expression for measured distance, expressed in meters, would be a scalar quantity:

$$d = \mid P_1 (t_1) - P_2 (t_1) \mid,\tag{9}$$

where $P_1(t_1)$ and $P_2(t_1)$ represent the locations of two different bodies at the same time. Measurements of distance are made, for example, in the center-to-center separation of engine cylinders and microelectronic circuit elements and in the calibration of the spacings of the successive graduations of various types of what are usually called "length scales," ranging from hundred-meter-long survey tapes to micrometer-long "pitch" standards for optical- and electron-microscope measurement systems.

## 2.2.2.4  Extension

**a.** *Geometrical Concept of Extension:*  In geometry, extension is the line segment between two points which lie on a surface which is topologically equivalent to a sphere such that all points on the line segment near the surface are either interior to or exterior to the surface. Equivalently, extension is the line segment connecting two points on a surface which have exterior normals with components that are anti-parallel.  A formally explicit expression for physical extension $e$  is:

$$e = \mid r_{b1}(\psi) - r_{b2}(\psi+180°) \mid, \tag{10}$$

where  $r_{b1}(\psi)$ and $r_{b2}(\psi+180°)$ represent the locations of points on a surface having exterior normals 180° apart.

**b.** *Physical Concept of Extension:*  In physics, extension is the amount of physical space that is occupied by, or enclosed by, a material object.  A formally explicit expression for physical extension is:

$$E = \mid R_{b1}(\psi) - R_{b2}(\psi+180°) \mid, \tag{11}$$

where $R_{b1}(\psi)$ and $R_{b2}(\psi+180°)$ represent respectively the locations of the opposite-facing boundaries of the physical object which, for convenience, may be expressed in terms of a vector radius from the centroid of the body.

**c.** *Measurement Concept of Extension:*  In measurements, extension is the assigned numerical value of the length of path between the location of one boundary of a material object relative to the location of another opposite-facing boundary, expressed in terms of the standard unit of measure.  A formal expression for measured extension is:

$$e = \mid P_{b1}(\psi) - P_{b2}(\psi+180°) \mid \tag{12}$$

where e is the extension and $P_{b1}$ and $P_{b2}$ represent the locations of opposite-facing boundaries of an object.  Extension-type dimensions of objects include height, width, diameter, and also "length."  Measurements of extension are made in general to achieve physical fit between mating parts.  Prototypical examples of extension measurements are those in the automotive industry wherein the outside diameter of piston rings and the inside diameter cylinder walls are measured to insure slip fit. For analogous reasons, extension measurements are also made in the microelectronics industry on critical elements of microcircuit devices.  Table 2 below summarizes key aspects of the various types of dimensional measurements just described.

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**Table 2.** Description of the quantity and nature of the length of path to be measured for each of the four dimensional measurement types

|  | Dimension | Quantity | The Length of Path Is That Between: |
|---|---|---|---|
| Position | Vector **P** | The location in space of a single object and an origin of coordinates (equivalent to a second, reference object) |
| Displacement | Vector **D** | The change in location of one object from time $t_1$ to time $t_2$ |
| Distance | Scalar d | The locations of two objects |
| Extension | Scalar e | The locations of opposite-facing boundaries of an object |

In familiar terms, each type of "length" relates to a different, but interrelated, concern. For example, the "length" of an automobile (i.e., its extension) describes how big a parking space is needed, the "length" of a racetrack (i.e., its distance) describes how far it is between start and finish lines, and the "length" of a stage's travel (i.e., its displacement) describes how far its carriage can move.

## 2.3 Basis of Dimensional Measurements

What "lengths" of all scales and types have in common is that each is measured and expressed in terms of a unit of length. For that unit to be the internationally standard unit of length, a formal definition must be operationally realized and transferred.

### 2.3.1 Realization of the Unit of Length

The function of a unit of length is to provide a metric for the physical space in which dimensional measurements are to be made. The function of a standard unit of length is to provide a metric which can be used in common in international science and trade.

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## 2.3.1.1  Formal Definition of the Meter

By treaty, the standard of length used in international science and trade is the meter. The meter is part of the International System of Units (SI) and is referred to as the fundamental, or SI, unit of length. Since 1983, the formal SI definition of the meter as the unit of length has been:

> *"The meter is the length of path travelled by light in vacuum in the time interval 1 / 299 792 458 of a second"* [22].

No longer based on lines scribed on a platinum-iridium "meter" bar (as it was from 1887 until 1960), the meter today is the distance which light travels in free space in (approximately) three billionths of a second.

## 2.3.1.2  Derived Nature of the Meter

The new definition of the meter makes it in effect a derived quantity governed by the length-time relation:

$$L = c \cdot T , \tag{13}$$

where L is the defined unit of length, c is the defined constant speed of light, and T is the defined unit of time. The second, the SI unit of time, is defined as a fixed number of counts of the frequency of oscillation of specific atomic states of a cesium-beam atomic clock.

## 2.3.1.3  Operational Realization of the Meter

The meter is operationally realized through a chain of intercomparisons of the frequencies of various microwave-, infrared-, and visible-wavelength lasers, based on the relationship of the frequency of electromagnetic waves and their wavelength:

$$f \cdot \lambda_o = c , \tag{14}$$

where f is frequency, $\lambda_o$ is vacuum wavelength, and c is value of the speed of light (that is, $2.99792458 \cdot 10^8$ m/sec). Based on this relationship, the ultimate limit on the uncertainty associated with the realization of the meter is that of the cesium clock, which is $10^{-13}$. At present, for dimensional measurements access in principle to the frequency cesium-beam atomic clock is through a chain of intercomparisons of wavelengths of microwave, infrared and visible sources by which the frequency of visible-wavelength lasers is determined.

## 2.3.2 Transfer of the Metric to Physical Objects

In practical dimensional measurements, the means by which the SI unit of length as the metric of physical space is transferred to measurements of physical objects -- from automobile piston rings to microelectronic circuit elements -- is through a system which successively involves visible light of known wavelength, displacement interferometry using that light, coordinate measurement systems based on that interferometry, and measurements on physical objects using such coordinate measurement systems.

### 2.3.2.1 Visible Light of Known Wavelength

For dimensional measurements, the practical primary standard of length today is the vacuum wavelength of the 633-nm red-orange line of an iodine-absorption-stabilized helium-neon laser. One may realize the SI-unit-based wavelength, to a particular accuracy, by buying a commercial helium-neon laser stabilized to a corresponding degree.

### 2.3.2.2 Displacement Interferometry

The means by which propagating laser light, which defines the metric of physical space, is coupled to dimensional measurements in that space is through interferometric measurements of displacement. The prototypical displacement interferometer is the two-beam, Michelson interferometer. The Michelson interferometer consists of a laser light source, beam splitter, fixed reflector (either a plane mirror or retroreflector), moving reflector and detector. In this type of interferometer the translation of the moveable reflector, which must be parallel to the axis of propagation of the laser light, is measured in terms of whole and fractional numbers of fringes generated by the interference of the beams in the stationary and moving arms of the interferometer.

The typical displacement interferometer of today employs the heterodyne principal for measuring the change in optical phase between the two beams by having one frequency-shifted with respect to the other. A widely used commercial heterodyne interferometer employs a two-frequency laser and the displacement of a plane mirror or retroreflector is measured by means of two channels of detectors, doublers, counters, subtractors and a calculator [23].

By whatever means the phase difference between the reference and moving-mirror beams is measured, the operational link between the SI-unit-carrying wavelength of visible light and a displacement measurement is through the interferometer equation:

$$D = L_1 - L_2 = N\lambda_o / 2n_m \cdot \cos\theta, \tag{15}$$

where D is the displacement, $L_1 - L_2$ is the change in length (i.e., extension) of the interferometer cavity, $\lambda_o$ is the vacuum wavelength of the laser light used, N is the real number describing the counted integer-order and measured fringe-fraction of the interference, $n_m$ is the index of refraction of the medium, and $\theta$ is the angle between the optical axis of the interferometer and the direction of propagation of the incident light. In multi-pass fringe-fractioning interferometers:

$$D = M\lambda_o / 2 \text{ nm K} \cdot \cos\theta, \qquad (16)$$

where M is the counted integer-order and measured fringe-fraction of the interference, m is the number of passes of the light through the interferometer, and K is the number of electronic sub-divisions of the interference fringes (typically a power of two ranging from 16 to 1024 depending on the system).

For the high-resolution commercial heterodyne interferometer system described above, the limit of resolution, that is, the least count in displacement measurement is about $\lambda$ /400 or 1.5 nm. Under development now at NIST is a specially optimized heterodyne interferometer intended to realize a least count of 20 times higher, that is, $\lambda$ /8000 or 0.07 nm.

Note that, as described here, in a laser displacement-interferometry system, it is the value of the vacuum wavelength of the laser light which provides the metric of physical space as the fundamental unit of length. It is the overall interferometer system doing the subdivision of fringes which provides the "scale," that is, the metric-less subdivisions of that unit. In common usage, however, displacement-measuring devices, such as interferometer systems, are often called "scales" and perform both of the functions described here: subdividing the axes (that is, providing the scale for an axis) and carrying the metric (that is, providing the standard unit of length, relative to which the "scales" have somehow been appropriately calibrated).

## 2.3.2.3  Coordinate Measurement System

After the wavelength of laser light and displacement interferometry, the next link to dimensional measurements of physical objects is a coordinate measurement system. A coordinate measurement system is necessary for the measurement of "position," that is, the location of a single point in space.

To establish a coordinate system, one must establish the means for realizing the geometry for physical space, including: *axes* (one, two, or three depending on the dimensionality of the system desired; *an origin* (the null-vector point from which radiate the axes of the coordinate system); *a scale along each of the axes* (that is, the graduation of each axis into subdivisions); and *a metric* (that is, a single measure of distance in the space of the reference frame to which the graduations of the axes relate).

In modern, automation-oriented manufacturing, the means by which the geometry of the coordinate measurement system is realized and precision dimensional measurements are made, is the coordinate measuring machine (CMM). A coordinate measuring machine embodies a coordinate system by means of its essential functional components, which are its frame, scales, carriages and probes as outlined in table 3.

Today's high-performance three-dimensional coordinate measuring machines take on a wide variety of forms depending on the specific way in which each of these functional components is embodied. Frames can be of the bridge or cantilever-arm type. Scales can be ruled-glass optical encoders or enclosed-path laser interferometers. Carriages can be translating and rotating tables and moving bridges. Probes can be touch-fire or analog mechanical-contact types or capacitive, optical, or machine-vision non-mechanical-contact types.

**Table 3.** **The principal functional components of a coordinate measuring machine (as well as of a machine tool)**

| | |
|---|---|
| Frame | The frame is the geometry-generating structure, which is the means for embodying the origin, axes, and angular relationships which comprise a coordinate system. |
| Scales | The scales are the displacement-measuring devices, which are the means for realizing the metric-based graduations along each axis of the frame. |
| Probe | The probe is the sensor system, which is the means for linking the boundary of the object to be measured to the frame and scales; in a machine tool, the "probe" is a material-moving element rather than a material-locating element as it is in a measuring machine. |
| Carriage | The carriage is the complete motion-generating system, which is the means for translating the object to be measured relative to the probe, frame, and scales. |

Whatever particular form a coordinate measuring machine takes on, its fundamental function is to provide measurements of locations in space, that is, of position. However, just as a manufacturer wants to buy holes not drills, users of CMM's in manufacturing want measurements of dimensions of objects, such as those of their products, not coordinates of locations in space.

## 2.3.2.4  Physical Object to be Measured

The terminus of the series of linkages, which allow dimensional measurements on physical objects in terms of the SI unit of length, is the object itself.  Now there are two different kinds of features on objects corresponding respectively to the distance-type and the extension-type of dimensional measurements described above.

**a.** *Distance-Type Features of Objects*:  Distance-type features, include, for example, the center-to-center spacings of cylinders of an automobile or spacing of left-edges of successive microcircuit elements.  While, in fact, always to some degree extended, distance-type features are treated as extensionless points.  Thus, the distance between automobile cylinders is measured in terms of their centroids.  Similarly, the distance between the moon and the earth is center to center.  Measurements of distance type features are inherently, in effect, point to point.

**b.**  *Extension-Type Features of Objects*:  Extension-type features include, for example, the diameters of the automobile cylinders or the widths of the microcircuit elements.  This type of feature must be treated as made up of boundaries that must be approached from opposite directions as in calipering, in which one locates two points, one on each and measures the "distance" between them.  However, it is in the essential nature of extension measurements that these boundaries are opposite facing.  As will be seen below, it is the practical and theoretical difficulty in locating properly such boundaries that makes extension the most difficult type of dimensional measurement.

In sum, successful measurement of, for example, the diameter of an automobile cylinder or the width of a microelectronic element in terms of the SI unit of length requires dimensional measurements of each type in succession: displacement, position, distance, and, finally, extension.  The measure of success in this succession of measurements at each level and in total is accuracy achieved.

## 2.3.3  Standards-Imposed Limits to Accuracy

The accuracy in practical dimensional measurements depends on the availability of standards for the calibration of the measuring machines by which measurements are made.  Attainable accuracy is ultimately limited by national standards laboratories in measurements involving a realization of the meter, displacement interferometry, reference coordinate measuring machines, and calibrated artifacts.

## 2.3.3.1 Limit of Realization of the Meter

The ultimate limit to accuracy in practical dimensional measurements is the accuracy with which national standards laboratories can realize the definition of the meter. At present that limit is the uncertainty with which the visible wavelength of the iodine-stabilized HeNe laser is known, that is, $10^{-9}$ compared to the $10^{-13}$ of the atomic clock. The meter as the unit-of-length metric of that space can be accessed directly then through the vacuum wavelength of the HeNe laser. Table 4 below shows the effective wavelength and uncertainty associated with HeNe lasers stabilized by various means.

**Table 4.** Comparison of the value and associated fractional uncertainty of the vacuum wavelength of helium-neon lasers of various degrees of stabilization

| Wavelength Value | Uncertainty | Type of Stabilization |
|---|---|---|
| 632. 991  nm | $10^{-6}$ | Free-running |
| 632. 9914 | $10^{-7}$ | Any one of a number of |
| 632. 99139 | $10^{-8}$ | opto-electronic techniques |
| 632. 991398 | $10^{-9}$ | Iodine Absorption Cell |

As indicated in table 4, the fractional uncertainties ( $\Delta\lambda/\lambda$ ) associated with commercially available HeNe lasers range from the $10^{-6}$ of a free-running device, through the $10^{-7}$ specified stability of certain commercial lasers, and $10^{-8}$ short-term stability of some such devices, up to the $10^{-9}$ accuracy of the visible wavelength of the iodine-stabilized laser as defined in the documents associated with the definition of the meter and realized in laboratory devices.

## 2.3.3.2 Limit of Displacement Interferometry

After the ultimate limit to accuracy of dimensional measurements imposed by the realization of the meter in terms of the wavelength of visible light, the next limit to that accuracy is the degree to which measurements of displacement can be made by means of optical-wavelength interferometry.

Based on work by national standards laboratories, the limit to the accuracy of interferometric displacement measurements at a displacement of a meter is estimated to be: $10^{-4}$ in air uncompensated for pressure, temperature, and humidity; $10^{-7}$ in fully compensated ambient air; and $10^{-8}$ in standard dry air. Based on results by a variety of workers, a reasonable estimate for the practical limit for displacement interferometry in vacuum is $10^{-10}$.

### 2.3.3.3  Limit of Coordinate Measurement

The next factor to limit the accuracy of dimensional measurements after the realization of the meter and displacement interferometry is the ability to embody a coordinate system in a measuring machine. At present, the highest-accuracy coordinate measurement is achieved by national standards laboratories on special-purpose one- and two-axis measuring machines. At the United States National Institute of Standards and Technology, a machine for the calibration of line scales over the range 1 µm to 1 m has associated with it a total uncertainty of measurement $U_T$ given by:

$$U_T = 10 \text{ nm} + 10^{-7} \cdot s_x ,\qquad (17)$$

where $s_x$ is the separation of any two graduations on the scale. At one meter, the positional error of this machine is approximately $1 \cdot 10^{-7}$ [24]. For a two-axis machine used for the calibration of grid plates up to 600 mm x 600 mm, the corresponding positional error in routine use is 0.5 µm, corresponding to about $5 \cdot 10^{-7}$ [25].

Currently unmet is the need in the microelectronic industry for an accuracy of 50nm accuracy over 250 mm, that is, $2 \cdot 10^{-7}$ [26]. Because of a trend to even more stringent requirements, for the regime of nanotechnology, there is under development at NIST a "Molecular Measuring Machine"[27], which is a planar 50-by-50 mm xy-coordinate measuring machine (with a 100 µm z-axis) which has a design-goal positional capability of:

$$U_T = 1 \text{ nm} = 10^{-8} \text{ @ } 70 \text{ mm}.\qquad (18)$$

### 2.3.3.4  Limit of Physical Artifact Standards

The limiting error in many practical dimensional measurements is the accuracy of artifact standards (that is, physical objects), dimensions of which have been calibrated by a national standards laboratory or by a secondary laboratory referenced to it. Accuracies representative of various types of artifact standards calibrated by NIST are shown in the table 5 below.

Note that because of their idiosyncratic nature and the intractability of dealing with the very object-specific boundary-location errors, NIST calibrates extension-type objects, such as integrated-circuit photomask linewidths [28], only when the national interest justifies the major long-term theoretical and experimental effort required.

**Table 5.** The type, range, and accuracies of some dimensional standards calibrated by the National Institute of Standards and Technology

| Dimensional Std | Type | Range | Accuracy |
|---|---|---|---|
| Survey Tapes | d | 1 mm to 300 m | 5.0 $\mu$m + $10^{-6} \cdot$ d |
| Gauge Blocks | d | 1 mm to 0.5 m | 0.025 $\mu$m to 0.15 $\mu$m |
| Line Scales | d | 1 $\mu$m to 1 m | 0.01 $\mu$m + $10^{-7} \cdot$ d |
| Photomask Lines | e | 0.5 $\mu$m to 20 $\mu$m | 0.05 $\mu$m |
| Polymer Spheres | e | 0.3 $\mu$m to 3 $\mu$m | 0.01 $\mu$m |
| Thin-Film Steps | d | 0.02$\mu$m to 10 $\mu$m | 0.003 $\mu$m to 0.2 $\mu$m |

Note also that while NIST provides measurement services for calibrations of distance and extension (which involve associating the unit of length with locations on physical objects), it does not calibrate displacement or position (which involve associating the unit of length with a device-based process [29].

## 2.4 Assessing Dimensional Measurements

As suggested earlier, manufacture of modern products demands dimensional measurements of not only exceptional precision, that is, the closeness together, but also of exceptional accuracy, that is, closeness to a true value [30].

Especially in high-technology products including, for example, microwave resonators for communication systems and x-ray optics for microelectronic lithography, accuracy is required because the dimension of device features is dictated not just by design convention but by the laws of physics. As a result, dimensions of features must not only be very close to *a* value but to *the* value. This section looks at the assessment of dimensional measurements in terms of statistical characterization of error, formal-theoretical characterization of error, compounding of error, and standards-imposed limits of error.

## 2.4.1 Statistical Characterization of Error

Physical measurement is itself a process, like manufacturing, which produces output. The output of a measurement process is numbers, that is, the measurement results. The quality of these numbers can be assessed and characterized by means of measurement statistics in terms of precision and accuracy. The manufacture of modern products demands dimensional measurements that are of both high precision and high accuracy, however one chooses to describe that combined state.

### 2.4.1.1 Precision vs. Accuracy

**a. *Precision*:** As a concept, precision conveys the notion of closeness together. As such, it is the principal figure of merit for the achievement of higher quality in the sense of lower variability. Precision is conventionally represented numerically in terms of the standard deviation from the mean for a number of measurements presumed to be randomly drawn from a large population describable by a Gaussian distribution function. The standard deviation for a single such measurement $\sigma_i$ is given by:

$$\sigma_i = [ \ \Sigma_i \ (\ell_i - \ell_m)^2/(n - 1) \ ]^{1/2}, \tag{19}$$

where $\ell_m = \Sigma_i \ell_i/n$ and the summations $\Sigma_i$ are over the number of measurements n. Where $\sigma_i$ is a measure of the precision of one measurement, $\sigma_m$ is a measure of the precision of the mean of n measurements and is given by:

$$\sigma_m = \sigma_i \ / \ n^{1/2}. \tag{20}$$

Precision is given in terms of either one-, two-, or three-sigma values for single measurements, corresponding respectively to confidences of 68.3%, 95.5%, and 99.7%. Historically national standards laboratories typically have used $3\sigma_m$ for characterizing dimensional standards which they calibrate, while instrument manufacturers often used $2\sigma_m$ for characterizing dimensional-measurement devices which they sell.

**b. *Accuracy*:** As a concept, accuracy conveys the notion of closeness to a true value. As such, accuracy is the principal figure of merit for the achievement of interchangeability of components and physical operation in high-technology devices. In those situations, actual not relative dimensions determine functional performance. Accuracy is conventionally represented numerically in terms of an estimate of the maximum that a measurement would likely be different from a true or standard value. In metrological terms, accuracy is the estimate of the degree to which measurements are free from systematic error and is represented by the term s.e.

### 2.4.1.2 Error or Total Uncertainty

The metrological term for a combined measure of precision and accuracy is uncertainty or error. Total uncertainty includes a fully-described estimate of systematic error and fully described measure of precision. NIST, along with a number of the national standards laboratories of other countries, up until the present have typically used the three-sigma of the mean in the assignment of a total uncertainty given by $U_T = \text{s.e.} + 3\sigma_m$.

Obviously, informed judgment must be made in how to sum and combine systematic and random error contributions. For example, the magnitudes of systematic errors may be added algebraically, that is, as signed quantities, if the signs are definitely known or arithmetically, that is, without signs, if not. Random errors may be added in quadrature if known to be uncorrelated or arithmetically if not. Throughout this chapter, the term "error" is used synonymously with uncertainty $U_T$ as described above and is denoted by the Greek character delta, upper case ($\Delta$) for total-type and lower ($\delta$) for type-specific [1].

### 2.4.1.3 Additive-Multiplicative Representation

Dimensional-measurement errors from different sources can be either additive in nature, that is, appearing as incremental off-set, or they may be multiplicative, that is, appearing as a length proportional. Total error can be of the form:

$$\Delta \ell = A + B \cdot \ell, \tag{22}$$

where A is the additive error and B the multiplicative.

---

[1] *Note Added at Time of This Report*: The approach to the expression of uncertainty recommended by the International Bureau of Weights and Measures (BIPM) which has been recently adopted by major international standards organizations is to: associate with each component of uncertainty a standard deviation of either Type A, evaluated by statistical procedures, or Type B, evaluated by other means; combine these uncertainty components in a sum of squares; and, if desired, to multiply by a coverage factor k, typically having a value from 2 to 3. As described in the appendix of section 4, the U.S. National Institute of Standards and Technology has now adopted the BIPM approach. Unless documented user's needs dictate otherwise, the "accuracy" of NIST measurements will be given on a $2\sigma$ basis, i.e., the estimated uncertainty will be a combined uncertainty with a coverage factor of 2.

**a.** *Added and Multiplicative Components*:  Given a system in which errors from a number of sources are to be combined to estimate the total error in a length-based dimensional measurement of a given type, then:

$$\Delta \ell = (\Sigma \delta \ell_{Ai}) + (\Sigma \delta \ell_{Bi}) \cdot \ell, \tag{23}$$

where A now is the sum $\Sigma_i$ over individual additive contributions $\delta \ell_{Ai}$ and B the sum $\Sigma_i$ over individual multiplicative contributions $\delta \ell_{Bi}$, where $A_i$ and $B_i$ can be comprised of either or both systematic errors and random errors.

**b.** *Manufacturers' Statements of Accuracy*:  Many commercial producers of measuring machines conventionally quote an accuracy for the performance of their products in additive-multiplicative form, as in, for example:

$$\Delta L = 2 \ \mu m + [ \ L(mm)/500mm \ ] \ \mu m, \tag{24}$$

which means that over the 1 m-travel of the machine, the accuracy of single measurements of position would range from a minimum of 2 $\mu$m to a maximum of 4 $\mu$m.

## 2.4.2  Formal-Theoretical Characterization of Error

Simple assessments and statements of accuracy which put errors into the simple additive-multiplicative form lump two types of additive errors, constant off-sets and non-linearities, and provide an inadequate basis for identification of the sources of error.  For diagnostic purposes, the better approach is to consider the most general case, that of a dimensional measuring system that produces output which includes linear and non-linear terms in the form:

$$\ell_{obs} = a + (1+b) \cdot \ell_t + c(\ell_t^n), \tag{25}$$

where $\ell_{obs}$ is the measured value, $\ell_t$ is the true length, a and b are constants and $c(\ell_t^n)$ is the sum of all non-linear terms. The error in measured values $\Delta \ell$ is then given by:

$$\Delta \ell = \ell_{obs} - \ell_t \ = \ a + b \cdot \ell_t + c(\ell_t^n). \tag{26}$$

Each of the error components a, b, and c describes a specific type of error source and is a measure of the degree to which the measurement system has realized a fundamental requirement of measurement.

## 2.4.2.1  Fundamental Axioms of Measurement

For dimensional measurements to yield physically meaningful results, they must conform to axiomatically fundamental requirements given in the *Philosophical Foundations of Physics* by Rudolf Carnap [31] and provided a more accessible account by Simpson [32]. These axiomatic requirements are that a measuring system must be able to rank order objects along the dimension of measurement and to reproduce properly the unit, the zero, and the scale of that dimension.

**a.  *The Rank-Order Operator*:**  The rank-order operator is the overall procedure by which objects or processes being measured are ordered and assigned values of the quantity being measured. In length-based dimensional measurements, the ordering operator is equivalent to a comparison of two objects involving, in effect, a translation in physical space. Each type of dimensional measurement, however, requires its own particular realization of that ordering operator. As a result, displacement, position, distance, and extension each constitutes its own measurement dimension which must be operationally tied to the dimension of physical space.

**b.  *The Unit*:**  The *unit* is the rank-order greater of two objects or states of the physical system assigned a defining numerical value of the quantity being measured. The unit is one of the two points required to specify what is axiomatically a linear system. In length-based dimensional measurements which conform to international standards, the unit is the SI unit of length. In practical dimensional measurements today, the unit of length is an internationally accepted value for a well-defined wavelength of an iodine-stabilized HeNe laser as discussed above in section 2.3.1.

**c.  *The Zero*:**  The *zero* is the rank-order lesser of the two objects or states of the physical system assigned a defining numerical value of the quantity being measured. In effect, the zero is the other of the two points required to specify the linear system. Implied by the definition of the unit of length is that the *zero* of measurement has an assigned numerical value of zero (that is, $|\ell_z| \equiv 0$) and that the zero of length of dimensional measurements is the length of the path in space which light traverses in the zero interval of time.

In practical dimensional measurements, the *zero* corresponds to the physical zero-vector origin of coordinates which must be practically realized in each of the different dimensional types. For example, in laser interferometry, the zero of displacement is the initial optical-path difference between the reference beam and moving-mirror beam, one which must remain constant during subsequent translation of the mirror in order to prevent loss of origin of displacement. In coordinate measuring machines, the zero of position is the initial location of the tip of the probe relative to the interferometer reflector, which must remain constant as the probe is moved about to prevent loss of origin of position.

**d.** *The Scale*: The scale is the set of equal graduations into which the difference between the unit and the zero is divided. In effect, the scale is the condition of linearity under which *differences* in dimensions are equal, that is, the difference between one pair of objects is the same as the difference between another pair. Operationally, the scale is the means for measuring in increments smaller than the unit.

In practical dimensional measurements, the *scale* is the combination of a physical transducer and a linearization algorithm by which linear interpolations between known points can be carried out. For example, in displacement interferometry, the scale is generated by interpolating between fringes using an assumed sinusoidal function of waveforms for the linearization algorithm. Non-linear transducers such as capacitance gauges -- when used in conjunction with suitable mathematical models of their responses -- can also be used to generate the required linear scale.

In sum, the axioms of measurement define the requisite elements of a measurement system, each of which must be operationally realized to achieve meaningful results. The first axiom -- the rank-order operation -- defines the dimension of measurement, such as temperature, time, voltage, or, in this case, each of the four types of length. The other three axioms are the means by which numerical values are associated with measurements of that dimension. table 6 below shows explicitly the form of the axiom errors for each of the dimensional types.

**Table 6. The matrix of errors by measurement axiom and dimensional type in general form**

| Dimensional Type | Error of the Zero | Error of the Unit | Error of the Scale |
|---|---|---|---|
| Displacement | $a_D$ | $b_D \cdot D$ | $c_D(D^n)$ |
| Position | $a_P$ | $b_P \cdot P$ | $c_P(P^n)$ |
| Distance | $a_d$ | $b_d \cdot d$ | $c_d(d^n)$ |
| Extension | $a_e$ | $b_e \cdot e$ | $c_e(e^n)$ |

## 2.4.2.2 Analysis of Error in Terms of the Axioms

Associated with each of the latter three axioms of measurement is a specific type of error with specific, fixable causes. Errors of the unit, the zero, and the scale correspond respectively to the terms in eq (26) with $a$ equal to the error of the zero, $b$ to the error of the unit, and $c(\ell_t^n)$ to the error of the scale, that is, the sum of all non-linear terms.

**a. *Error of the Zero*:** Errors of the zero occur for measurement systems which have a properly linear relation of measured value to true value, $\ell_{obs}$ to $\ell_t$, but have a non-zero value at $\ell_t = 0$, that is:

$$\ell_{obs} = \ell_t + a \tag{27}$$

and
$$\Delta \ell_z = a.$$

An error of the zero arises, for example, due to a shift in the origin of coordinates in the course of a dimensional measurement.

**b. *Error of the Unit*:** Errors of the unit occur for measurement systems which -- while having a properly linear relation of measured value to true value -- have a non-unitary slope, that is:

$$\ell_{obs} = (1+b) \cdot \ell_t \tag{28}$$

and
$$\Delta \ell_u = b \cdot \ell_t.$$

Error of the unit corresponds, for example, to an erroneous value of wavelength in displacement interferometry or a constant misalignment of coordinate and object axes.

**c. *Error of the Scale*:** Errors of the scale occur for measurement systems which have a non-linear relation of measured value to true value, that is:

$$\Delta \ell_s = c(\ell_t^n). \tag{29}$$

Error of the scale represents failure of the interpolation scheme to divide the difference between the unit and the zero into strictly equal intervals, that is, the degree to which the system is nonlinear. The measurement system is linear, if given four objects (i.e., four displacements, positions, distances, or extensions) for which the true difference between pairs is the same, the measured differences are also the same. Error of the scale is, by definition, the inequality in those differences, that is:

$$\Delta_s = ( \ell_{obs1} - \ell_{obs2}) - ( \ell_{obs3} - \ell_{obs4} ) \tag{30}$$

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when
$$( \ell_{t1} - \ell_{t2} ) = ( \ell_{t3} - \ell_{t2} ) .$$

That the error of the scale corresponds to a non-zero value of the coefficient $c(\ell_t^n)$ can be seen when the non-linear response of a dimensional measurement system is given a specific form, such as the quadratic:

$$\ell_{obs} = a + (1+b) \cdot \ell_t + c\ell_t^2 . \tag{31}$$

Evaluation of eq (30) at four values of $\ell_t$, denoting the differences in the pairs of true values by $\Delta_i$, and substitution into eq (26) leads to the result:

$$\Delta \ell_s = c \cdot (\ell_{t1} - \ell_{t3}) \cdot \Delta_i . \tag{32}$$

An error of the scale arises, for example, from to faulty interpolation of fringes in interferometry or uncompensated nonlinearities in LVDT or capacitive displacement measuring devices.

**d.** *Errors of Zero, Unit, and Scale*: In summary, the axiom-specific errors for the general case that $\ell_{obs} = a + (1+b) \cdot \ell_t + c(\ell_t^n)$ are given by:

$$\Delta \ell = \Delta \ell_z + \Delta \ell_u + \Delta \ell_s = a + b \cdot \ell_t + c(\ell_t^n) \tag{33}$$

where: a corresponds to $\Delta \ell_z$, the error in realizing the zero; b corresponds to $\Delta \ell_u$, the error in realizing the unit; c corresponds to $\Delta \ell_s$, the error in realizing the scale; and $\ell$ is any one of the four dimensional measurement types.

## 2.5  Principal Specific Sources of Errors

Errors in dimensional measurements are particular to each of the different types and, since each succeeding type is dependent on that which precedes it, total error increases as one proceeds from the displacement, position, and distance to the extension of objects.

### 2.5.1  Error in Displacement Interferometry

The type of dimensional measurement most directly linked to the SI unit of length and the basis for all subsequent types of dimensional measurements is displacement by laser interferometry, that is, the measurement of a linear change of location in space with time of a single object, in this case, the moving mirror of the interferometer. The measure of the limit of this ability is displacement uncertainty or error, $\delta D$. Errors in measurement of displacement by laser interferometry are associated with each of the terms in eq (15).

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## 2.5.1.1 Error of Wavelength in Vacuum $\lambda_o$

The uncertainty in the wavelength in vacuum of the laser light depends on the type of stabilization used in its construction. As indicated in table 4 above, uncertainty in the vacuum wavelength of the 633 nm red-orange line of the helium-neon (HeNe) lasers widely used in dimensional metrology range from $10^{-6}$ for unstabilized lasers down to $10^{-10}$ for iodine-stabilized ones.

## 2.5.1.2 Error of Index of Refraction of Medium n

The uncertainty in the wavelength in medium of the laser light depends on the index of refraction of the medium through which the light propagating in the interferometer passes.

**a.** *Error of the Index of Ambient Air*: Since index of refraction of a gas is a function of its temperature, pressure, humidity, and composition, without compensation for actual variations in those parameters the upper limit of uncertainty for interferometric displacement measurements in ordinary ambient air can be large: for example, an index error of $10^{-6}$ would result from any one of the following variations: a 1 degree Centigrade change in temperature, a 0.33 kPa (2.5 mm Hg) change in atmospheric pressure, or an 80% change in relative humidity [33].

**b.** *Error of the Index of Compensated Standard Air*: Errors associated with the index of refraction of the ambient medium can be reduced by compensation achieved by measurement of the index of the actual ambient air or by calculation of the index of standard dry air. The Edlen formula is an internationally agreed upon equation for the calculation of the index of refraction of standard air as a function of wavelength, air temperature, air pressure, and relative humidity [34].

With compensation, the lower limit of uncertainty for practical laser displacement interferometry in ambient air is estimated to be about $1.2 \cdot 10^{-7}$ [35]. The uncertainty in the Edlen formula itself for the index of refraction of standard dry air is estimated to be $5 \cdot 10^{-8}$ [36]. Operation of an interferometer in vacuum eliminates this source of error.

**c.** *Error of Index of Refraction in the Dead Path*: So-called "dead path" error is due to improper compensation for the difference in the lengths of the optical paths of the two interfering beams at the zero-displacement position of the system when index-altering environmental changes occur during the course of the displacement measurement. For commercial systems which use sensor-based index compensation, dead-path errors are largely compensated for in software, leaving a residual error of $1.4 \cdot 10^{-7}$ times the dead-path distance.

### 2.5.1.3  Error in Fringe Fractioning

Errors in realizing a scale by means of displacement interferometry arise from various sources in the process of the generation, subdivision, and counting of the interference fringes. Representative of such errors are those which arise in widely-used displacement interferometers of the polarization beam-splitting type. The principal sources of error in this type of system are those associated with electronic subdivision, polarization mixing, thermal drift, and dead path [37].

**a.**  *Error in Electronic Subdivision*:  The inherent half-wavelength resolution of interferometers, corresponding to the spacing of the alternating light and dark of the fringes, can be extended by a variety of electronic fringe-interpolation schemes, each of which has its own limiting resolution which contributes an additive, least-count error. Operating at 633 nm, certain commercial polarization heterodyne interferometer systems have least counts in electronic-subdivision, depending on whether used with retro-reflectors or plane mirrors, of $\lambda/32$ (approximately 20 nm) and $\lambda/64$ (approximately 10 nm) respectively.

**b.**  *Error From Polarization Mixing*:  Less-than-perfect separation of the polarization states of the interfering beams in polarization-type interferometers, due to leakage of one component into the other, produces a non-linear error in displacement measurement with such systems. Such error varies as a function of change in optical path with a periodicity of the wavelength of the laser source with an amplitude specific to an individual interferometers. For one commercial linear interferometer system, the peak-to-peak phase error was found to be 5.4° corresponding to approximately 5 nm.

**c.**  *Error From Thermal Drift*:  Changes in temperature within the optical components of the interferometer system, which produce a differential change in path optical length between the interfering beams, give rise to drift type errors in displacement measurement. This environmentally induced error is less for temperature-controlled systems and those which are specifically designed to deal with this source of error. An example of the latter has a quoted thermal error of 40 nm/C°, which is 12 times better than a conventional plane-mirror system with its 0.5 μm/C° [12].

### 2.5.1.4  Error in Alignment of Interferometer

Finally, constant angular misalignment between the interferometer and the incident light gives rise to an error (derived from the basic interferometer equation):

$$\delta D_\theta = D \cdot (-\theta^2/2), \tag{34}$$

where $\delta D_\theta$ is misalignment contribution to the displacement error, D is the measured displacement, and $\delta\theta$ is the small angle of misalignment between the axis of interferometer

(the normal to the parallel faces of the interferometer mirrors) and the axis of propagation of the incident laser light.

In sum, this section has looked at sources of errors in the type of dimensional measurement most directly linked to the SI unit of length, those of laser displacement interferometry by the best available form of that technique, polarization-type heterodyne interferometry.

## 2.5.2 Error in Coordinate-Position Measurement

After displacement, the type of dimensional measurement next most directly linked to the SI unit of length and inherent in subsequent types is position, that is, measurement of the location of a single point relative to a coordinate system. The measure of the limit of the ability to measure position is position error, $\delta P$. Errors in position measurements are associated with each of the functional components of the coordinate measurement system as embodied by a coordinate measuring machine: the scales, frame, carriage, and probe.

## 2.5.2.1 Error in Relation to Displacement Scales

Lack of commonality of origin and co-linearity of axes of the coordinate-position measurement system and the displacement system, which supplies the metric and scale subdivisions, gives rise to two types of errors specific to positional measurement.

a. *Misalignment of Mirror Translation*: Constant angular misalignment between the axis of the displacement measurement and the axis of the coordinate system gives rise to "cosine error," which is governed by the relationship:

$$P = D / \cos \alpha \tag{35}$$

where D is the displacement, P is the coordinate position being assigned by the measurement and $\alpha$ is the angle of misalignment between the two. For a displacement generated by an interferometer system, $\alpha$ is the angle between the axis of translation of the moving mirror and the optical axis of the interferometer (i.e., the normal to the parallel faces of the reference and moving mirrors). For a small angle, the cosine error contribution to coordinate-position error, a multiplicative error, is given by:

$$\delta P_c = D \cdot ( -\alpha^2/2). \tag{36}$$

Cosine error arises, in effect, when the axis of the displacement vector and axis of the coordinate vector are rotated with respect to each other but have a common origin. [Note that the errors due respectively to misalignment of interferometer and light (eq (34)) and misalignment of mirror translation and interferometer (eq (36)) are of the same sign while

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that due to misalignment of object and mirror translation (eq (40)) is opposite (appendix, section 2.8).]

**b.** *Abbe Error*: Abbe error can arise when the axes of the displacement vector and the coordinate vector do not share a common origin *and* they rotate with respect to each other, that is, tilt, during the course of the displacement. Abbe error is governed by the relationship:

$$D = P + O_a \cdot \sin \phi \qquad (37)$$

where D is the measured displacement, P is the coordinate being assigned by the measurement, $O_a$ is the lateral distance between the displacement and coordinate axes (called the Abbe offset), and $\phi$ is the angle of tilt which occurs during the course of the displacement measurement.
For small angles, the Abbe error contribution to coordinate-position error, which is an additive, is given by:

$$\delta P_a = O_a \cdot \phi. \qquad (38)$$

## 2.5.2.2  Error Related to Carriage Motion

Carriage-associated position-measurement error occurs in measuring machines in which the motion-generating system (which translates the object relative to the reference frame) is the same as the displacement-measuring system (which measures that translation) as, for example, in a machine which uses the same lead screw both to move a carriage and to measure its position. For such machines, carriage-associated errors include backlash and hysteresis, both of which are direction-dependent and result in loss of origin of the coordinate system.

## 2.5.2.3  Errors Related to Reference-Frame Geometry

Limitations in the mechanical structure of a coordinate measuring machine give rise to positional errors associated with the machine's coordinate axes and their angular relationships. Since the reference frame of a coordinate measuring machine must consist of one highly planar reference surface for each coordinate axis with each of these planes angularly oriented with respect to the others in a constant and well-known way, the errors associated with reference frame geometry are complex.

Characterization of the positional errors in a 3D CMM requires dealing with 21 degrees of freedom based on 6 degrees per axis (3 translational plus 3 angular, including roll, pitch, and yaw) plus the three angular orientations of the axes with respect to each other. Specialized

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matrix-based models and measurement algorithms for characterizing positional errors in coordinate measuring machines and machine tools have been developed [38,39] and standardized techniques adopted [40].

### 2.5.2.4 Error Related to the Probe

Limitations in the mechanism for linking the object to be measured to the machine's coordinate measurement system give rise to positional errors specifically associated with the machine's probe. Probe-specific errors are of two types, depending on whether the probe is designed to carry out only functions of a probe as a simple sensor or a probe as a measuring machine.

**a.** *Positional Error of Probe-as-Sensor*: Positional errors of a probe-as-sensor arise to the degree that it fails to carry out its primary function, that of detecting -- in a binary sense of "here" or "not here" -- the boundary of an object, such as to locate it relative to the coordinate system of the machine. Positional errors of probe-as-sensor include what in touch-trigger mechanical-contact probes is called "pre-travel," that is, the displacement of the probe tip after contact with the object but before the probe signals that contact. Such error arises, for example, from variations in bending of the probe structure and the threshold response of the mechanical-to-electrical transducer.

**b.** *Positional Error of Probe-as-Measuring-Machine*: Positional errors of a probe-as-measuring-machine arise in, for example, "analog probes," which are designed not only to carry out the primary function of linking machine and object, but also to provide an output signal proportional to the displacement of the probe tip on contact with the object. Positional errors of analog probes, being as they are miniature measuring machines attached to a larger machine include: 1) errors of the probe as a sensor; 2) errors of the probe as a measuring machine (including those associated with scale, frame, and carriage); and 3) errors associated with the orientation of the probe as one coordinate system with respect to another.

In sum, this section has looked at sources of errors in measurements of position, which after displacement, is the next most directly linked to the SI unit of length. As indicated, position error deals with the ability to measure location of points in space and are associated with each of the functional components of a coordinate measuring machine by which such position measurements are made.

### 2.5.3 Error in Distance Measurement

After displacement and position, the type of dimensional measurement next most directly linked to the SI unit of length is that of distance, that is, measurement of the separation of

two successively located point-like features on material objects. The measure of the limit of the ability to measure distance is distance error, $\delta d$. Errors specific to measurements of distance are associated with the probing, angular alignment, and environmental-dependence of separation of distance-type features of the object.

### 2.5.3.1  Distance Error From Feature Probing

Errors of feature probing arise from variations in the size, shape, and material of the individual point-, line-, and plane-like features, the separation of which is being measured in a distance measurement. Such variations limit the ability to reproducibly locate features

with a probe. Errors associated with such variations are assessed in terms of the repeatability with which the probe of a particular coordinate measuring machine can be set on a particular feature.

### 2.5.3.2  Misalignment of Object and Coordinate Axes

Misalignment between the object being measured, and a coordinate axis of the measuring machine gives rise to cosine error governed by the relationship:

$$d = |P_1 - P_2| \cdot \cos \gamma, \qquad (39)$$

where $P_1$ and $P_2$ are the measured coordinate positions of the point-like features, d is the distance being measured, and $\gamma$ is the angle of misalignment between the coordinate axis and that of the object itself. For a small angle, the cosine error contribution to distance error is given by:

$$\delta d_c = d \cdot (+\gamma^2/2). \qquad (40)$$

### 2.5.3.3  Error From Object Environment

Apart from the effects of changes in the environment on displacement-measuring devices and the measuring machine itself, such environmental changes give rise to errors associated with the object itself, including those associated with thermal expansion, mechanical distortion, and contamination.

**a.** *Error From Object Thermal Expansion*: Distance error due to thermal expansion of the object can be due to different temperatures for the whole object or different temperatures for two positions on the object. In either case, variation in the dimensions of the object with temperature is governed by the equation:

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$$\rho = -(\Delta \ell / \ell_0) / \Delta T, \tag{41}$$

where $\rho$ is the coefficient of linear thermal expansion of the material and $\Delta \ell / \ell_0$ is the fractional change in the "length" of material for a difference in temperature $\Delta T$. Note that since thermal expansion is defined in terms of the *change* in length (either between distance-type features or extension-type boundaries), it is inherently a distance-type property given by:

$$\delta d_T = \rho \cdot \Delta T. \tag{42}$$

**b.** *Error From Object Distortion*: Distance error can also arise from any other factor which, in effect, distorts the object by altering the relative separation of features of the object. Such factors include any local or overall mechanical distortion of the object due, for example, to fixturing of the object on the measuring machine; and any gain or loss of material on one feature with respect to the second due, for example, to contamination or wear.

In sum, this section has looked at sources of errors in measurements of distance which, after displacement and position, is the next most directly linked to the SI unit of length, such errors being associated with measurement of the separation of two point-like features of material objects.

## 2.5.4 Error in Extension Measurement

The type of dimensional measurements operationally farthest removed from the primary realization of the SI unit of length is that of extension. The measure of the limit of the ability to measure extension is extension error, $\delta e$. Most specific to the measurement of extension-type features of objects, such as the inside diameters of holes and the outside diameters of plugs, is error associated with locating a single boundary and error associated with locating one boundary relative to another one.

## 2.5.4.1 Error in Location of a Single Boundary

**a.** *Error in Reducing a 3D Surface to a Point*: Errors occur in measuring the location of a real boundary because a material object is not an ideal geometrical surface being intersected by a line such as to define a point. Instead that boundary is a rough three-dimensional surface being contacted by a finite-area probe. Estimates of the contribution to boundary-location errors for specific object-probe combinations may be made based on measurements of surface roughness, which involves statistical averaging over surface irregularities. Boundary-location errors due to surface roughness can be comparable to the roughness itself, which for a highly-polished steel surface, for example, can be of the order of 0.05 μm.

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**b.** *Error in Compensation of Probe-Object Interaction*: Errors occur in measuring the position of a single boundary as a result of any uncompensated off-set between the location of the probe and the location of the surface which the probe signals. For example, in locating the boundary of a machined part with a mechanical stylus, single-boundary-location errors occur due to incomplete compensation for both: the finite radius of the probe and its finite penetration into the part. Analogous errors occur in locating the boundary of a microcircuit element with the beam of an electron microscope.

Compensation for probe-object interaction, such as the deformation of a part by a mechanical stylus and penetration into the bulk of a material by an electron beam, can only be done by fundamental theoretical modelling of the specific probe and object involved. Of necessity, such modelling has been done, for example, for high-accuracy optical- and electron-microscope calibrations of photomask linewidth standards for use in the microelectronics industry [41].

In sum, error in boundary location is a vector having a magnitude (associated with, for example, the finite roughness of the surface, the finite extent of the probe and the finite depth of penetration of the object by the probe) and a direction, positive or negative, relative to the axis of coordinate measurement.

## 2.5.4.2 Error in Location of Two Boundaries

In measurements of extension, errors in the location of one boundary combine as vectors with errors in the location of the second boundary -- that is, one of opposite orientation -- and the results are non-canceling, as indicated by:

$$\delta_e = \delta P_{b1} + \delta P_{b2}, \tag{43}$$

where $\delta P_{b1}$ is the error in the location of the first boundary and $\delta P_{b2}$ is the error in the location of the second.

For the case that the boundary-location error is the same for the two boundaries, that is, where the magnitude of $\delta P_{b1}$ equals that of $\delta P_{b2}$, the error in extension measurement due to the combination of the two boundary-location errors is:

$$\delta_e = \mp 2\delta P_b, \tag{44}$$

where the minus sign is for outside-caliper measurements (as in the diameter of a plug) and the plus sign is for inside-caliper measurements (as in the diameter of a hole) when the boundary-location errors are due, in effect, to "penetration" (positive or negative) of the object by the probe.

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An example of a positive boundary-location error is that due to the finite radius of a mechanical probe (that is, the extension of the probe tip along the axis of measurement). An example of a negative boundary-location error is probe pre-travel (that is, the finite displacement of the probe along the axis of measurement after mechanical contact with the object, but before the probe transducer crosses a threshold and signals).

Additive boundary-location errors are unique to extension-type measurements. In measurements of displacement, position, and distance, the boundaries involved face in the same direction and boundary-location errors cancel, that is:

$$\delta P_{D,P,\text{or } d} = \delta P_b - \delta P_b \equiv 0. \tag{45}$$

## 2.5.5 Compounding of Error in Measurement Types

A final aspect of assessing accuracy in practical dimensional measurements involves a look at the cascading nature of the errors in the succession of the dimensional measurement types. This cascading, that is, accumulation, of errors is shown formally below, where $\Delta$ represents the total uncertainty of the type of measurement and $\delta$ represents the type-specific error in that total.

In this notation, the total error in displacement measurement $\Delta D$ is a function of $\delta D$, which is all the displacement-specific errors described in section 2.5.1, plus $\Delta \lambda_o$, which is the error specific to the metric-carrying vacuum wavelength of laser light source, and is given by:

$$\Delta D = \Delta D(\delta D, \delta \lambda_o). \tag{46}$$

Similarly, the total error in position measurement $\Delta p$ a function of $\delta p$, which is all the position-specific errors described in section 2.5.2, plus $\Delta D$, which is the total error in displacement measurement, and is given by:

$$\Delta p = \Delta p(\delta p, \Delta D) \tag{47}$$

$$= \Delta p(\delta p, \delta D, \delta \lambda_o).$$

And again, the total error in distance measurement, $\Delta d$, a function of the total error in position measurement, plus all the distance-specific errors described in section 2.5.2 above, and is given by:

$$\Delta d = \Delta d(\delta d, \Delta p) \tag{48}$$

$$= \Delta d(\delta d, \delta p, \delta D, \delta \lambda_o)$$

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Finally, the total error in extension measurement, $\Delta e$, a function of the total error in distance measurement, plus the extension-specific errors described in section 2.5.4 above, and is given by:

$$\Delta e = \Delta e(\delta e, \Delta d) \tag{49}$$

$$= \Delta e(\delta e, \delta d, \delta p, \delta D, \delta \lambda_o)$$

As is indicated by this formal analysis, the accuracy of each type of dimensional measurement depends on - and is limited by - the total error of the preceding type, as well as by the sum of its own type-specific errors.

In sum, measurements of extension are the most difficult in which to achieve accuracy because they inherit all the errors of the successive displacement, location, and distance measurements upon which they are based and, addition, are subject to intractable boundary-location errors to which the others are not.

## 2.6  Error Budget Example

All of the considerations of accuracy assessment discussed above can be taken into systematic account in the assessment of the errors in a particular dimensional measurement system by compilation of an error budget for the measuring system in question. In this example, a simple sum-over-errors is used to estimate the total worst-case error; other means for estimating the most likely errors have been described [42].

The example considered here is measurement of an extension-type object on a laser-interferometer-based single-axis coordinate measuring machine. In this example, the overall system consists of a stabilized HeNe laser, a commercial heterodyne interferometer system, a machine frame with a moving carriage, a touch-fire probe, and a 100 mm long steel rod, the length of which is to be measured, all within a normal atmospheric environment.

Table 7 below shows a compilation of the contributions to the overall accuracy of measurement of the specified object with the hypothetical coordinate-measuring-machine

**Table 7.** Error budget example: analysis of measurement of extension of a 100 mm long object with an interferometer-based coordinate measuring machine

| Error Contribution by Source | | $\delta\ell_{AI}$ | | $\ell_{BI}$ |
|---|---|---|---|---|
| Interferometer Least Count | $\delta N_{lc}$ | 0.005 μm | | |
| Beam-Polarization Mixing | $\delta N_{pm}$ | 0.002 μm | | |
| Machine-Path Atmosphere | $\delta n_{pa}$ | | | 10 ppm |
| Compensated | $\delta n_{pc}$ | | | 0.15 ppm |
| Optics Thermal Drift | $\delta n_{io}$ | 0.02 μm | | |
| Dead-Path Compensated | $\delta n_{dp}$ | 0.015 μm | | |
| D-vs-λ Alignment | $\delta\theta_{to}$ | | | 0.05 ppm |
| | | | | |
| Sum of D-Specific Errors | $\delta D$ | 0.042 μm | | 0.20 ppm |
| Vacuum-Wavelength Error | $\delta\lambda_o$ | | | 0.02 ppm |
| <u>Displacement Error Total</u> | $\underline{\Delta D}$ | <u>0.042 μm</u> | ± | <u>0.22 ppm</u> |
| | | | | |
| P vs. D Alignment | $\delta P_\phi$ | | | 0.05 ppm |
| Carriage Abbe Tilt | $\delta P_\beta$ | 0.24 μm | | |
| Probe Setting | $\delta P_p$ | 0.50 μm | | |
| | | | | |
| Sum of p-Specific Errors | $\delta P$ | 0.74 μm | | 0.05 ppm |
| Displacement Error Total | $\Delta D$ | 0.042 μm | | 0.22 ppm |
| <u>Position Error Total</u> | $\underline{\Delta p}$ | <u>0.782 μm</u> | ± | <u>0.27 ppm</u> |
| | | | | |
| O vs. P Alignment | $\delta d_\gamma$ | | | 1.5 ppm |
| Thermal Expansion | $\delta d_\rho$ | | | 1.0 ppm |
| Point-Feature Definition | $\delta d_s$ | 0.10 μm | | |
| | | | | |
| Sum of d-Specific Errors | $\delta d$ | 0.10 μm | | 2.5 ppm |
| Position Error Total | $\Delta p$ | 0.782 μm | | 0.27 ppm |
| <u>Distance Error Total</u> | $\underline{\Delta d}$ | <u>0.882 μm</u> | ± | <u>2.77 ppm</u> |
| | | | | |
| Arbitrarily-Located Bndry | $\delta e_{bx}$ | 1.0 μm | | |
| Compensated | $\delta e_{bt}$ | 0.1 μm | | |
| Shape-Averaged Boundary | $\delta e_{bm}$ | 0.2 μm | | |
| | | | | |
| Sum of e-Specific Errors | $\delta e$ | 0.3 μm | | |
| Distance Error | $\Delta d$ | 0.882 μm | | 2.77 ppm |
| <u>Extension Error Total</u> | $\underline{\Delta e}$ | <u>1.182 μm</u> | ± | <u>2.77 ppme</u> |

system described. The organization of the error budget follows the sequence of the type scheme used throughout the chapter. In the table, $\delta \ell_{Ai}$ and $\delta \ell_{Bi}$ represent, respectively, additive and multiplicative contributions to the type-specific errors. The values are estimates of the relative size of error contributions from the various sources. In the following sections, each of the individual contributions to the type-specific errors for this hypothetical example are discussed in turn.

### 2.6.1  Error in Displacement Measurement $\Delta D$

In this example the errors in the laser-interferometric measurement of the displacement derive from displacement-specific errors associated with each of the parameters in the basic interferometer equation plus the error associated with the vacuum wavelength of the laser light.

### 2.6.1.1  Fringe-Fraction Error $\delta D_N$

The contributions to displacement-specific measurement error are due to errors in determination of the change in the order of interference with the translation of the interferometer reflector:

$$N = i + f, \tag{50}$$

where N is a real number which is the sum of the counted order i and measured fringe fraction f. Practically, the total error in N is associated with error in interpolating between fringes, that is:

$$\delta N = \delta f. \tag{51}$$

In this example, the contributors to error in fringe-fractioning are the 5 nm due to the least count of interferometer and the 2 nm due to the mixing of the polarization states of the interfering beams.

### 2.6.1.2  Index-of-Refraction Error $\delta D_n$

Contributions to displacement-specific measurement error arise from variations in the indices of refraction along the optical paths of the interfering beams due to changes in environmental conditions. The variation in phase between the two beams as a function of variations in the individual indices of refractions in each of the segments of the reference- and moving-arm beams is given by:

$$\delta \Delta \phi = (2\pi/\lambda_o) \left[ \Sigma_i \, \delta n_{ri} \cdot \ell_{ri} - \Sigma_i \, \delta n_{mi} \cdot \ell_{mi} \right], \tag{52}$$

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where $\Sigma_i$ indicates a sum over the i number of optical-path segments $n_i \cdot \ell_i$ in each of the arms. In this example, the principal segments of the overall optical path include the CMM machine-axis path along which the moveable reflector is translated, the dead path, and the interferometer-optics path, that is, the glass through which the light passes.

**a.** *Machine-Path Atmosphere*: The first and major source of index-of-refraction errors is the variation of the atmospheric air along the displacement path. The following show the changes in temperature, pressure, and relative humidity, each of which gives rise to a displacement error of $10^{-6}$:

$$
\begin{aligned}
\delta n_{\Delta T} &= & 1 \text{ ppm} / 1 \,^\circ\text{C} \\
\delta n_{\Delta P} &= & 1 \text{ ppm} / 0.33 \text{ kPa } (2.5 \text{ mm Hg}) \\
\delta n_{\Delta H} &= & 1 \text{ ppm} / 80\% \,\Delta\text{RH}.
\end{aligned}
\tag{53}
$$

In this example, a $\pm 0.5\,^\circ$C change in temperature, a $\pm 25$ mm change in pressure, and a $\pm 10\%$ change in relative humidity combine to produce a multiplicative error of about 10 ppm. With measurement of the changes and compensation for them, this is reduced to 0.15 ppm.

**b.** *Interferometer-Optics Thermal Drift*: The second source of index of refraction error is change in the temperature of the glass components of the interferometer through which the light in the two beams passes. In this example, optics thermal drift error is 0.02 $\mu$m.

**c.** *Dead-Path Compensation*: A last source of index of refraction error is change in temperature, pressure, and humidity of the dead path (the difference in the interferometer arms at the starting point of the machine-axis path). In this example, compensation has reduced dead-path error to 0.015 $\mu$m.

## 2.6.1.3 Interferometer-Axis Alignment Error $\delta D_{\theta ti}$

The contribution to displacement-specific measurement error due to the angular misalignment $\theta_{ti}$ between the axis of the interferometer and the axis of propagation of the incident light is, for a small angle of misalignment, given by:

$$
\delta D_{\theta ti} = D \cdot (-\,\theta_{ti}^2/2).
\tag{54}
$$

For an angular alignment corresponding to a lateral displacement of one-tenth the spot size of a 3 mm beam at the 1 meter travel of the measuring machine stage, $\theta_{ti} = 3\times10^{-3}$ radians and the resulting error $\delta D_{\theta ti}$ is 0.05 ppm.

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## 2.6.1.4  Sum of Displacement-Specific Errors $\delta D$

In this example, the total displacement-specific error $\delta D$ is the sum of the errors due to fringe-fractioning, index of refraction, and angular misalignment of translation-propagation axes and is given by:

$$\delta D \quad = \quad \Sigma_i \, \delta D_i$$

$$= \quad (\Sigma \delta D_{Ai}) \quad + \quad (\Sigma \delta D_{Bi}) \cdot \ell \qquad (55)$$

$$= \quad 0.042 \; \mu m + 0.20 \; ppm \cdot D \; .$$

At a displacement of 100 mm (corresponding to the extension of the object being measured), the sum of the displacement-specific errors is:

$$\delta D_{100} \quad = \quad 0.062 \; \mu m \quad = \quad 0.62 \; ppm \; @ \; 100 \; mm \quad (56)$$

## 2.6.1.5  Vacuum-Wavelength Error $\delta \lambda_o$

The contribution to the total displacement measurement error $\Delta D$ due to the error in the value of the vacuum wavelength of the laser is:

$$\delta D_{\lambda o} \quad = \quad (\delta \lambda_o / \lambda_o) \cdot D \; , \qquad (57)$$

which for the highly stabilized commercial laser of this example is $2 \cdot 10^{-8}$.  Thus,

$$\delta D_{\lambda o} \quad = \quad 0.02 \; ppm \cdot D. \qquad (58)$$

## 2.6.1.6  Total Displacement Error $\Delta D(\delta D, \delta \lambda_o)$

As indicated, combining the displacement-specific error $\delta D$ with the error in the vacuum-wavelength of the laser source $\delta \lambda_o$ yields the total displacement error $\Delta D$ over the 1 meter travel of the measuring machine stage, that is:

$$\Delta D \quad = \quad \delta D + \delta \lambda_o$$

$$= \quad (\Sigma \delta D_{Ai}) + (\Sigma \delta D_{Bi}) \cdot \ell \; + \; (\delta \lambda_o / \lambda_o) \cdot \ell \qquad (59)$$

$$= \quad 0.042 \; \mu m + 0.22 \; ppm \cdot D$$

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For a displacement measurement corresponding to length of the 100 mm long object, the total error in the displacement measurement is:

$$\Delta D \quad = \quad 0.064 \ \mu m \quad = \quad 0.64 \ ppm \ @ \ 100 \ mm. \quad (60)$$

## 2.6.2 Error in Position Measurement $\Delta P$

In this example the errors in the coordinate-measuring-machine measurement of position derive from errors associated with the geometrical relationships among the scale, frame, carriage and probe of the CMM and the setting ability of the probe itself.

### 2.6.2.1 Misalignment of Coordinate and Displacement

The contribution to position-specific measurement error due to the angular misalignment $\phi$ between the axis of translation of the mirror and the axis of the interferometer is, for a small angle, given by:

$$\delta P_\phi \quad = \quad D \cdot (- \phi^2/2). \quad (61)$$

For an angular misalignment of $\phi_{ti} = 3 \cdot 10^{-3}$ radians, the resulting error is:

$$\delta P_\phi \quad = \quad 0.05 \ ppm \cdot P. \quad (62)$$

### 2.6.2.2 Carriage Abbe Tilt

The contribution to position-specific measurement error due to a difference in angle of tilt of a non-zero-length probe between two positions (Abbe error) is, for a small angle:

$$\delta P_a \quad = \quad O_a \cdot \beta. \quad (63)$$

In this example, for a distance between the axis of the probe and the axis of the displacement measurement of 50 mm and for a change in tilt of 1 arc second, the Abbe error contribution to coordinate-position error, which is an additive, is:

$$\delta P_a \quad = \quad 0.24 \ \mu m. \quad (64)$$

### 2.6.2.3 Probe Setting

The contribution to position-specific measurement error due to probe setting is a measure of the ability of an individual probe to reproducibly locate unvarying, uni-directionally

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approached, probed features. Since a location measurement requires the probe to trigger twice (once at the location of the origin and once at the location being measured, an estimate of the error in probe-setting based on the $2\sigma$ setting of a good-quality commercial probe operated in the touch-trigger mode is:

$$\delta P_p \qquad = \qquad 2 \cdot 0.25 \ \mu m = 0.50 \ \mu m. \qquad (65)$$

### 2.6.2.4 Sum of Position-Specific Errors $\delta p$

In this example, the total position-specific error $\delta P$ is the sum of the errors due to coordinate-scale misalignment, Abbe error, and probe setting.

$$\delta P \qquad = \qquad \Sigma \ \delta P_i$$

$$= \qquad 0.74 \ \mu m + 0.05 \ ppm \cdot d \qquad (66)$$

$$= \qquad 0.745 \ \mu m \ @ \ 100 \ mm$$

### 2.6.2.5 Total Position Error $\Delta P(\delta P, \Delta D)$

Combining the position-specific errors, $\delta p$, with the total displacement error, $\Delta D$, over the 1 meter travel of the measuring machine stage yields:

$$\Delta P \qquad = \qquad \delta P + \Delta D \qquad = \qquad 0.782 \ \mu m + 0.27 \ ppm \cdot d \qquad (67)$$

For a position corresponding to length of the 100 mm long object, the total error in the position measurement is:

$$\Delta P_{100} \qquad = \qquad 0.809 \ \mu m \qquad = \qquad 8.1 \ ppm \ @ \ 100 \ mm \qquad (68)$$

### 2.6.3 Error in Distance Measurement $\Delta d$

In this example the errors in the coordinate-measuring-machine measurement of distance derive from distance-specific errors associated with the object alignment, temperature, and distance-type features, plus the error in position measurement.

### 2.6.3.1 Object-Axis/Probe-Path Alignment

The contribution to distance-specific error due to the angular misalignment $\gamma$ between the axis of the object and the axis of coordinate measurement, that is, the path of the probe, is, a small angle, given by:

$$\delta d_\gamma \quad = \quad d \cdot ( + \gamma^2/2). \tag{69}$$

For an angular misalignment of 0.1° (corresponding to a lateral distance of less than 0.2 mm over the 100 mm length of the object):

$$\delta d_\gamma \quad = \quad 1.5 \text{ ppm} \cdot d . \tag{70}$$

Note that this error due to misalignment of object and coordinate axis (eq (40)) is opposite in sign to that due to misalignment of interferometer and light (eq (34)) and misalignment of mirror translation and interferometer (eq (36)).

### 2.6.3.2 Object Thermal Expansion

The contribution to distance-specific error due to the difference in temperature $\Delta T$ between the object to be measured and a reference temperature (such as that of the CMM) is determined by the linear thermal expansion coefficient $\rho$ for the object according to the thermal expansion equation:

$$\delta d_{\Delta T} \quad = \quad \rho \cdot \Delta T \cdot d \tag{71}$$

For the a steel object in this example, $\rho$ is 10 ppm/°C, which for temperature difference $\Delta T$ of 0.1 °C gives:

$$\delta d_{\Delta T} \quad = \quad 1.0 \text{ ppm} \times d. \tag{72}$$

### 2.6.3.3 Object Point Definition

The contribution to distance-specific measurement error due to object point definition is a measure of the ability to associate with the object features which can be characterized as points, the position of which points can be reproducibly located by a variability-free probe. In this example, the estimate of a practical lower limit for error associated with locating points on real objects (twice that for defining a single point) is estimated to be:

$$\delta d_s \quad = \quad 0.10 \ \mu m. \tag{73}$$

### 2.6.3.4 Sum of Distance-Specific Errors $\delta d$

In this example, the total distance-specific error $\delta d$ is the sum of the errors due to coordinate-scale misalignment, Abbe error, and probe setting.

$$\delta d \qquad = \qquad \Sigma \; \delta d_i = \qquad 0.10 \; \mu m + 2.5 \; ppm \cdot d. \qquad (74)$$

At a distance of 100 mm corresponding to the extension of the object being measured, the sum of the distance-specific errors is:

$$\delta d_{100} \qquad = \qquad 0.35 \; \mu m \qquad = \qquad 3.5 \; ppm \; @ \; 100 \; mm. \qquad (75)$$

### 2.6.3.5 Total Distance Error $\Delta d(\delta d, \Delta p)$

Combining the sum of the distance-specific errors, $\delta d$, with the total position error, $\Delta p$, over the 1 meter travel of the measuring machine stage yields:

$$\Delta d \qquad = \qquad \delta d + \Delta p \qquad = 0.882 \; \mu m + 2.77 \; ppm \; x \; d. \quad (76)$$

For a distance corresponding to the length of the 100 mm long object, the total error in the distance measurement is:

$$\Delta d \qquad = \qquad 1.109 \; \mu m \qquad = \qquad 1.1 \; ppm \; @ \; 100 \; mm. \qquad (77)$$

### 2.6.4 Error in Extension Measurement $\Delta e$

In this example, the errors in the coordinate-measuring-machine measurement of extension derive from extension-specific errors associated with boundary location and definition, plus the error in distance measurement.

### 2.6.4.1 Boundary-Location Errors

Boundary-location error in extension measurements represents the systematic failure of the probe to signal the proper location of the material boundary of the object and is of the form:

$$\delta_e \qquad = \qquad \mp 2 \cdot \delta P_{bx}, \qquad (78)$$

where the minus sign is for outside-caliper measurements, the plus sign is for inside-caliper measurements, and $\delta P_{bx}$ is positive for location of the arbitrary boundary behind the material boundary.

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**a.** *Location of Arbitrary Boundary*: Various phenomena give rise to the direction-dependent boundary-location errors inherent to extension measurements which individually contribute to the aggregate effect and $\delta P_{bx}$. In this example, there is the probe radius (that is, the finite extension of the probe tip along the axis of measurement) and the probe pre-travel (that is, the finite displacement of the probe along the axis of measurement after mechanical contact with the object is made, but before the probe transducer crosses a threshold and signals). Assuming that the gross effects of probe radius and pre-travel are eliminated by pre-characterization of the probe, there remain the extension-specific sources of error due, for example, to deformation of the object by mechanical contact and variation of pretravel of the probe with the direction of approach to the object.

Given a three-dimensional touch-trigger probe with a specified pre-travel variation over 360° in the X-Y plane of ± 0.5 μm, the extension-specific error is taken to be:

$$\delta e_{bx} \qquad = \qquad 2 \cdot 0.5 \ \mu m \qquad = 1.0 \ \mu m. \tag{79}$$

**b.** *Compensated Boundary Location*: The contribution to extension-specific error due to location of an arbitrary boundary can in some situations be compensated by an empirical procedure while in the majority of cases a theoretical model of the probe-object interaction is required. In the example being followed here, the probe is assumed to have a residual direction-dependent variation in pretravel of:

$$\delta e_{bt} \qquad = \qquad 0.1 \ \mu m. \tag{80}$$

## 2.6.4.2 Average-Material-Boundary Location

Extension-specific error due to average-boundary location is associated with local variation in the position of the material boundary of the object such that it has no single extension. In the example being followed here, the steel block is assumed to have a surface-finish peak-to-valley of 0.1 μm (0.067μm RMS) which when mechanically probed leads to a contribution to extension-specific error:

$$\delta e_{bt} \qquad = \qquad 0.2 \ \mu m. \tag{81}$$

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### 2.6.4.3  Sum of Extension-Specific Errors δe

In this example, the total extension-specific error $\delta D$ is the sum of the errors associated with mislocation of a detected boundary relative to a material boundary and mislocation of a material boundary relative to some simple geometrical shape used to describe the object.

$$\delta e \quad = \quad \Sigma\ \delta e_i \ = \ \delta e_{b_t} + \delta e_{b_m} \tag{82}$$

$$= \quad 0.2\ \mu m + 0.1\ \mu m = 0.3\ \mu m$$

Note that extension-specific error is a purely additive error, independent of the magnitude of the extension.

### 2.6.4.4  Total Extension Error Δe(δe,Δd)

Combining the sum of the extension-specific errors $\delta e$ with the total distance error $\Delta d$ yields the total extension error:

$$\Delta e \quad = \quad \delta e + \Delta d \quad = \quad 1.182\ \mu m + 2.77\ ppm \cdot d \tag{83}$$

For the 100 mm long object, the total extension error, computed and rounded to a significant figure, is:

$$\Delta e_{100} \quad = \quad 1.459\ \mu m$$

$$\rightarrow \quad 1.5\ \mu m = \ 15\ ppm\ @\ 100\ mm. \tag{84}$$

### 2.6.5  Summary and Analysis of Errors

This error-budget analysis -- carried out for the case of a commercial-quality displacement-interferometer-equipped coordinate measuring machine used to measure the length of a polished, parallel-faced, 100 mm long, steel block -- illustrates issues specific to the example as well as ones associated with precision dimensional measurements in general.  The specific numerical results may be summarized and then analyzed in terms of their relative type-hierarchical, additive-multiplicative, and axiom-specific behaviors.

## 2.6.5.1  Summary of Numerical Results of Analysis

Table 8 below summarizes results specific to the hypothetical example being examined. Tabulated are errors specific to the type and total errors for each type, on both an incremental ($\pm\delta\ell$) and a fractional ($\pm\delta\ell/\ell$) basis.  For comparison, also shown (in italics) is the error in the vacuum wavelength of the laser which provides the reference to the SI unit of length.

**Table 8.  Summary of total errors by dimensional type in the example of the laser-interferometer-based CMM measurement of extension of a 100 mm block**

| Dimensional Type | Type-Specific Error | | Total-Error in Type | |
| --- | --- | --- | --- | --- |
| | Add $\delta\ell$ | Mult $\delta\ell/\ell$ | Add $\Delta\ell$ | Mult $\Delta\ell/\ell$ |
| Extension | 0.3 μm | ------ | 1.462 μm | 14.62 ppm |
| Distance | 0.35 μm | 3.5 ppm | 1.102 μm | 11.02 ppm |
| Position | 0.75 μm | 7.5 ppm | 0.812 μm | 8.12 ppm |
| Displacement | 0.06 μm | 0.6 ppm | 0.062 μm | 0.62 ppm |
| *Wavelength* | ------ | ------ | *0.002 μm* | *0.02 ppm* |

## 2.6.5.2  Analysis of Results

The particular results of the CMM/extension-measurement example illustrate a number of more general patterns in dimensional measurements including: a hierarchy of accuracies, dominance by additive errors and multiplicative errors at the short end and long end of the range respectively, and the minimal contribution of laser-wavelength error to total error.

**a.  *Hierarchy of Accuracies Among the Types*:**  The last column, which gives the total error $\Delta\ell/\ell$ by type, illustrates the inherent loss of accuracy as one moves through the progression of types of dimensional measurements.  Note that the accuracy of the displacement measurements is down by an order of magnitude from that of the reference wavelength. The accuracy of position is down by another order from there, while the accuracy of distance and extension down another factor of 2 or 3 further from there.  These trends in these relationships are general.  On the extremes, displacement measurements always have the best and more readily obtained accuracy and position and distance are intermediate, while extension measurements always have the lowest accuracy and the most dearly bought.

**b.** *The Additive-Multiplicative Errors*: The second and third columns, which give the type-specific errors, $\delta\ell$ and $\delta\ell/\ell$ respectively, illustrate the additive vs. multiplicative character of errors in the different types. On the extremes, the extension-specific errors are purely additive, which means they dominate at short lengths. In contrast, the displacement-specific errors are almost purely multiplicative, which means they dominate at long ranges. For the hypothetical CMM of the example, these trends are further illustrated by table 9 which shows errors in ranges of 10 and 1000 mm. Note especially in the table the near-pure additive 1.2 $\mu$m (121 ppm) error in extension at 10 mm compared to the near-pure multiplicative 0.26 ppm (0.26 $\mu$m) error in displacement at 1000 mm.

**Table 9.** **Total error by dimensional type at ranges of 10, 100, and 1000 mm in the example of the laser-interferometer-based CMM measurement of extension of a block**

| Dimension | Error @ 10 mm | Error @ 100 mm | Error @ 1000 mm |
|-----------|---------------|----------------|-----------------|
| Extension | 1.2 $\mu$m /121 ppm | 1.5 $\mu$m / 15 ppm | 3.9 $\mu$m / 4 ppm |
| Distance | 0.9 $\mu$m / 91 ppm | 1.2 $\mu$m / 11 ppm | 3.6 $\mu$m / 3.6 ppm |
| Position | 0.08 $\mu$m / 8 ppm | 0.11 $\mu$m / 1 ppm | 0.35 $\mu$m / 0.35 ppm |
| Displcmnt | 0.04 $\mu$m / 4 ppm | 0.06 $\mu$m / 0.6 ppm | 0.26 $\mu$m/ 0.26 ppm |

## 2.6.5.3 Axiom-Type Matrix Analysis of Results for the CMM Example

The error-budget for the CMM measurement of the 100 mm block serves to illustrate the use of the axiom-type matrix of errors for the analysis of the performance of the overall system of machine, object, and techniques for the achievement of precision dimensions. Table 10 shows in the type vs. axiom matrix format of table 6 the results of the CMM-extension error budget summarized in table 7. Also shown, in italics, are examples of the matrix elements not illustrated by the CMM-extension example.

**a.** *Errors in the Zero of Each Type:* As indicated in table 10, each of the types of dimensional measurements in the CMM-block example has associated with it a type-specific error of the zero ($\delta\ell_z$) corresponding to a shift of the effective origin of that type. In the displacement measurement, there are the uncompensated change in the index of refraction of the dead path, the movement of the effective optical-path location of the interferometer reference due to thermal drift, and the uncertainty in the location of the zero due to the least-count of the interferometer system. In the position measurement, there are the non-

**Table 10.** The axiom-type error matrix for the example of the laser-interferometer-based CMM measurement of extension

| Dimension | Error of Zero | Error of Unit | Error of Scale |
|---|---|---|---|
| Displacement | Dead-Path<br>Thermal-Drift<br>Least-Count | Laser Wavelength<br>Mirror-Path Index<br>Cosine I vs. L | Polarization Mixing<br>*Grad in MP Index*<br>*Bending of MP Ways* |
| Position | Probe-Tilt Abbe<br>Probe-Setting | Cosine T vs. I | *Grad in Table Temp* |
| Distance | Point-Definition | Cosine O vs. T<br>Object T-Expansion | *Bending of Object* |
| Extension | Arbitrary Bndry<br>Matl-Av-Bndry<br>*Backlash* | *Thickness-Dependent*<br>*Probe "Penetration"* | *Approach to Probe*<br>*Resolution Limit* |

cumulative change in the location of the position origin due to the tilt of the probe relative to the displacement axis (Abbe error) and the uncertainty in its location due to variations in probe-setting. In the distance measurement, there is the uncertainty in objects geometry which limits reduction to centroids, that is, points. Finally, in the extension measurements, there are the arbitrariness of the boundary located relative to the material boundary and the variation of the location of the material boundary about its mean. While not in the example, backlash corresponds to an error of the zero of extension because it only appears with the bi-directional approach to the object.

**b.** *Errors in the Unit of Each Type:* Similarly, as indicated in table 10, each of the types has associated with it a type-specific error of the unit $(\delta \ell_u)$, that is, error proportional to the first order of the dimension measured. In displacement, there are the error in the value of the vacuum wavelength of the laser, in the index of refraction of the path of the moveable mirror, and the angle of misalignment between the axis of the interferometer (I) and the direction of propagation of the incident light (L). In position measurement, there is the angle of misalignment between the axis of translation of the mirror (T) and the axis of the interferometer (I). In distance measurement, there is angle of misalignment between the object (O) and the probe-path, which for a rigid-body system is the same as the axis of translation of the mirror (T). While not discussed in the example, an error in the unit of extension can also arise due to any dependence of the effective "penetration" of the probe (positive or negative) on the thickness of the object.

**c. *Errors in the Scale of Each Type:*** Finally, as indicated in table 10, for each dimensional type, there are type-specific nonlinearities corresponding to errors of the scale ($\delta \ell_{s1}$). In the displacement measurement, there is the intermixing of the reference- and measuring-beams due to their imperfect separation by polarization states which leads to a non-linearity in the interpolation between fringes. While not in the example, other conditions lead to errors of the scale. In displacement measurements, these include a gradient in the index of refraction along the interferometer mirror path (MP) as well as any lateral bending of the mechanical ways which support the mirror along that path; in position measurements, a gradient in the temperature of the part of the machine which supports the object to be measured; in distance measurements a bending of the object; and in extension measurements, an approach to the resolution limit of the probe.

In sum, this analysis of the hypothetical case of the use of a laser-interferometer-based coordinate measuring machine for measurement of the length of a part has shown the nature and types of the errors that can arise in that specific example. What the example illustrates is that the use of the axiom-type matrix of errors provides as a complete, self-consistent scheme for systematically partitioning and analyzing such errors. The approach is, however, more generally useful.

## 2.6.5.4 Application of Type-Axiom Error Matrix to Manufacturing Operations

The axiom-type matrix just applied to the characterization of the dimension-measurement process applies equally to the characterization of the dimension-generation process, that is, the manufacture-to-design of a dimensionally specified product.

**a. *"Machine Tool" versus "Measuring Machine":*** Machine tools for the generation of dimensioned forms and measuring machines for the characterization of dimensioned forms share the elements of frame, carriage, scale, and "probe," where the former's probe is for changing the location of material and the latter's "probe" is for ascertaining where the material is located. This commonality is shared by all such "machine tools" — whether metal-cutting lathes, ion-beam milling machines, or photolithographic step-and-repeat cameras — and all such measuring machines — whether they be coordinate measuring machines, measuring microscopes, or micrometer calipers. As such machine tools and measuring machines must conform to the same axiom-type system to carry out their essential functions.

**b. *Example of Extension Errors in Milling:*** All the errors indicated in table 10 for the CMM measurement of a block are also manifest in machine-tool-based manufacture of dimensionally specified products.

Consider briefly, for example, the use of a laser-interferometer-based milling machine for the manufacture of a multiple-finned turbine-blade fan where the part design calls for the

machining of equally spaced fins of identical tapering cross-section. In general the machine will be subject to all the sources of error identified in table 10. Even under the conditions that the machine can generate a cutter path that conforms ideally to the part design (that is, can generate displacement, position, and distance perfectly), errors of extension can still occur.

Under certain conditions of tool-force and material-stiffness, deflection of the fin by the cutter will vary in proportion to the thickness of the fin at the point of cutting, resulting in error in the material form even for an errorless cutter-path. Thus, while the spacing between fins is perfect (corresponding to no errors of the unit, the zero and the scale in distance), the error in the cross-section of each fin would have a "cooling-tower" shape corresponding to errors in each of the zero, unit, and scale of extension and fully analogous to the arbitrary boundary, thickness-dependent probe-penetration, and limit-of-resolution errors indicated in table 10.

## 2.7 Conclusion

This chapter on precision dimensional measurements for modern manufacturing has presented a new scheme for the assessment of dimensional error, both in measurement processes and in manufacturing processes. The scheme is based on a matrix of dimensional types and measurement axioms which forms a complete, self-consistent system for assessing dimensional error. The scheme has been illustrated by a case study of a coordinate measuring measurement of an extended object, with the type-axiom related errors indicated as being similarly manifest in manufacturing processes such as the milling of turbine blades. This type-axiom matrix should provide a useful means for the exposure of error, the diagnosis of its cause, and the means of practical elimination through more effective design of machines, processes, and techniques for the manufacture of products with precisely dimensioned forms.

## 2.8 Appendix: Errors Due to Angular Misalignments of System Elements

Accuracy of measurement of dimensions of physical objects by means of a coordinate measurement system based on a laser displacement interferometer, such as the prototypical interferometer shown schematically in a plan-view representation here, depends upon precise alignment of the axes of the laser, interferometer, translating mirror, and the object.

Errors in measurements of displacement, position, and distance (and/or extension) arise respectively from misalignment of pairs of axes — interferometer and laser, translating mirror and interferometer, and object and translating mirror, as described individually and in combination below.

## 2.8.1 Misalignment Error Specific to Displacement

For light of wavelength $\lambda_0$ propagating in a medium of index of refraction n in a direction **K** at angle $\theta$ to the axis of the interferometer (defined by the normals **N** to the reference and moveable reflectors), the change in spacing of the interferometer mirrors, $\Delta s$, is related to the observed change in fringe order number, $\Delta m$, by the interferometry equation:
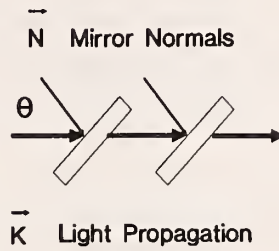


**Figure 5. Alignment of mirror normals and light direction.**
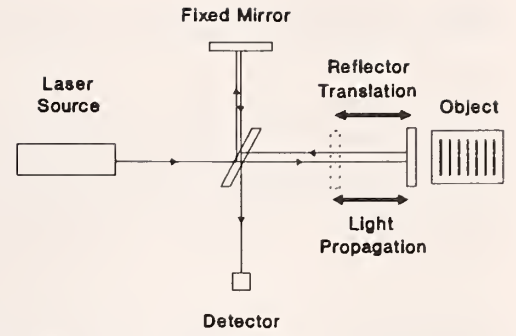


**Figure 4. Sources of misalignment errors in displacement-measuring interferometers.**

$$D = \Delta s = \Delta m \cdot \lambda_0/2n \cdot \cos\theta . \tag{A1}$$

Note that misalignment between the axis of the interferometer and the axis of propagation of the incident light constitutes an error of the metric or unit of displacement relative to that of the light.

## 2.8.2 Misalignment Error Specific to Coordinate Position

For a moveable or "measuring" reflecting surface (plane mirror or retroreflector) which translates at an angle $\phi$ relative to the axis **N** of the parallel-plate interferometer, the translation **T** is related to the change in spacing $\Delta s$ of the interferometer by the "cosine-error" equation:

$$P \cdot \cos\phi = T \cdot \cos\phi = \Delta s . \tag{A2}$$



**Figure 6. Alignment of mirror translation and mirror normals.**

Note that misalignment of the axis of translation of the measuring element of the interferometer relative the axis of the interferometer constitutes an error of the metric or unit of position (coordinate) relative to that of the displacement.

### 2.8.3 Misalignment Error Specific to Object-Distance

Finally, given an object with features with an axis $O$ oriented at an angle $\gamma$ relative to the direction of translation of the measuring mirror of the interferometer, the relation of the distance between features on the object and the translation of the mirror is given by the equation:

$$d = T \cdot \cos\gamma . \tag{A3}$$

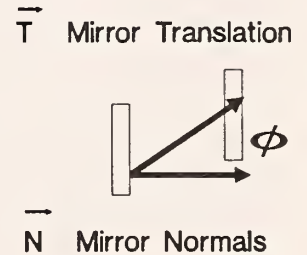Note that the misalignment of the axis of the object and that of the translation of the measuring mirror constitutes an error in the metric or unit of distance relative to that of position.

$\overrightarrow{T}$ Translation Axis
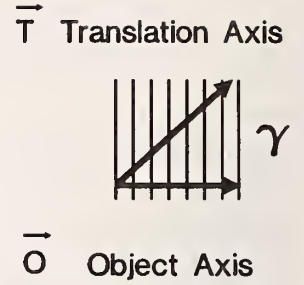


$\overrightarrow{O}$ Object Axis

**Figure 7. Alignment of part axis and mirror translation.**

### 2.8.4 Combined Misalignment Error in Distance

For a system in which there are all three misorientations described above, the relation of the distance between features on the object and the observed change in the fringe order number is given by:

$$d = (\Delta m \cdot \lambda_o \cdot \cos\gamma) / (2 \cdot n \cdot \cos\theta \cdot \cos\phi). \tag{A4}$$

### 2.8.5 Explicit Form for Misalignment Error

Given the sign convention that the observed distance $d_{obs}$ is the algebraic sum of the true distance $d_{true}$ plus the error term $\Delta d$, then:

$$\Delta d = d_{obs} - d_{true}$$

$$= d - d(\theta, \phi, \gamma) \tag{A5}$$

$$= (\Delta m \cdot \lambda_o/2n) \cdot [ 1 - \{\cos\gamma/(\cos\theta \cdot \cos\phi)\}]$$

Under the conditions that the angles $\theta$, $\phi$, and $\gamma$ are each small, successive substitution of the approximations $\cos x \approx 1 - x^2/2$ and $1/(1 - x^2) \approx 1 + x^2$ into eq (A5) leads to:

$$\Delta d \approx (\Delta m \cdot \lambda_o/2n) \cdot [ 1 - \{(1 - \gamma^2/2)/(1 - \theta^2/2) \cdot (1 - \phi^2/2)\}] \tag{A6}$$

$$\approx (\Delta m \cdot \lambda_o/2n) \cdot [ 1 - \{(1 - \gamma^2/2) \cdot (1 + \theta^2/2) \cdot (1 + \phi^2/2)\}] ,$$

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which with multiplication and retention of terms of the lowest order yields the final result:

$$\Delta d \approx (\Delta m \cdot \lambda_o/2n) \cdot ( + \theta^2 + \phi^2 - \gamma^2)/2 \qquad (A7)$$

Note that since the first and second terms are negative and the third term is positive, the overall error in distance due to angular misalignments can be either positive or negative and the measured distance can be either greater than or less than the actual.

## 3    A Comparison of National Standards for the Performance Evaluation of Coordinate Measuring Machines in Terms of Length-Based Dimensional Quantities

### 3.1    Introduction

A principal means to higher quality in the manufacture of products having dimensioned parts is higher accuracy in both the machines which form the parts and the machines which measure them.  The dimensions of parts are of different types, each comprising a different length-based dimensional quantity.

Since modern manufacturing depends on interchangeability of parts traded both domestically and internationally, accuracy in these various types of part dimensions consists not only of conformity of the dimensions to design but conformity of the units of the dimensions to a standard.  The international standards of length is the meter, which is defined in terms of the propagation of light in free space.  Practical realization of the formal definition is by means of laser interferometry and another length-based dimensional quantity, displacement.

This chapter compares U.S., German, British, and Japanese standards for evaluation of coordinate measuring machines and shows that using a variety of special artifacts the U.S. standard alone specifies separate tests for each of the four modes of length measurements (displacement, position, distance, and extension) and that, depending on the number and orientations of faces probed, a bi-directional step gage of the type suggested in the other standards can be used for tests of each of these.

Figure 8 is an engineering drawing representation of an automobile engine which illustrates some of the principal types of geometrical dimensions of discrete parts.  These types of geometric features include:   A, representing a length or width of the engine block; B, representing the diameter of a cylinder bore; C, representing the distance between the centers (axes) of cylinder holes; D, representing the depth of a blind hole; E,  representing the (true) position of a feature relative to the origin of a part coordinate system defined by, for example, reference holes in the oil pan plane; and F, representing the location of the center (axis) of a hole relative to a plane.
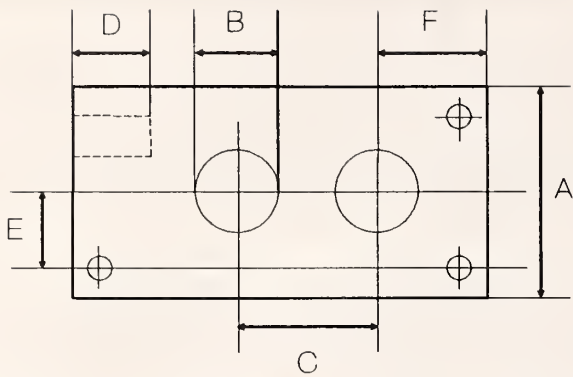
**Figure 8. An engineering drawing representation of the principal geometrical dimensions of a manufactured discrete part.**

Each of the various part features shown in figure 8 corresponds to a different type length-based measurement quantity: displacement, position, distance, and extension. For example, in this case of the automobile engine, there are extension-type features such as the length of the block and the diameter of cylinders; there are distance-type features such as the spacing of axes of the cylinders; and there are position-type features such as the locations of cylinders relative to reference features such as dowel holes. Features such as the depths of blind bore holes are most akin to a displacement. The significance of this approximate correspondence between types of features and types of length measurements is that in both the manufacture and measurement of dimensioned parts such as these, each of the length-based dimensional quantities involved is susceptible to type-specific errors which propagate and compound in a way which has been described in detail in chapter 2 [43] and outlined in the appendix (section 3.5).

For the evaluation of the accuracy of either machine tools or measuring machines, there are two general approaches relative to these part and length measurement types: first, overall evaluation in terms of a machine's ability to produce/measure part dimensions of the extension type; and evaluation of ability to produce/measure the aspects of each of the types — displacement, position, distance, and extension. Representative of the former approach is the German/European VDI/VDE standard for the evaluation of coordinate measuring machines; representative of the latter is the U.S. ANSI B89.1.12.

## 3.2 Standard-Based Performance Tests of CMM's

Two industry standards commonly referenced by U.S. manufacturers of commercial coordinate measuring machines are the U.S. ASME B89.1.12 [44] and German VDI/VDE 2617 [45]. Other major national standards for CMM's are the Japanese JIS B7440 [46] and the British BS 6808 [47].

### 3.2.1. U.S. Standard B89.1.12M

American National Standard ASME B89.1.12M-1990 specifies five principal tests for the evaluation of the dimensional measurement capabilities of CMM's. These five performance tests are called: Measuring Repeatability, Linear Displacement Accuracy, Volumetric Performance, Bi-Directional Length Measurement Capability, and Point-to-Point Probing
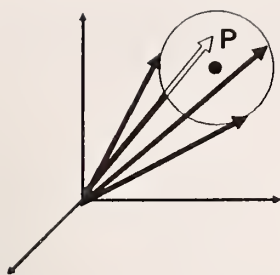
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Performance.

### 3.2.1.1 Measuring Repeatability

The first B89.1.12M machine performance test is measuring repeatability and deals with the capability of the CMM to properly measure position.

**Figure 9. B89.1.12M test for measuring repeatability.**

In this test a precision reference ball is located at the mid-point of travel of each of the machine's three linear axes. Four widely spaced non-planar contacts are made with the ball. From this set of four readings are calculated coordinates of the center of the ball. Ten determinations of these center coordinates are so made. The range of the ball center coordinate, that is, the maximum minus the minimum, is determined for each axis.

The measuring repeatability is, then, this spread in the calculated locations of the center of the ball for the 10 repeats. Thus this "measuring repeatability" performance test gives an estimate, on a per axis basis, of the variability in measurement of position at a single location:

$$\Delta P_x = |\, P_{xmax} - P_{xmin}\,| . \tag{1}$$

### 3.2.1.2 Linear Displacement Accuracy
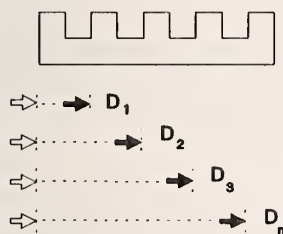
**Figure 10. B89.1.12M test for displacement accuracy.**

The second B89.1.12M machine performance test, linear displacement accuracy, deals in the words of the standard with the conformance of the machine's scales to the international standard of length.

In this test, either a step gage or a laser interferometer may be used. For each axis, the step gage or laser interferometer is oriented along a line parallel to that axis through the center of the work zone. Measurements are made for at least 10 equal intervals, which are specified to be greater than 25 mm and less than one-tenth the axis length. The accuracy for a given step is the difference between the step gage calibration or laser reading and the mean corrected machine reading. The linear displacement accuracy is then the sum of the magnitudes of the maximum positive and maximum negative errors.

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Thus this "linear displacement accuracy" performance test gives an estimate of the maximum error in the ability to measure from any location to any other location in the full travel and in the measurement of displacement-type features (see appendix, section 3.5):

$$\Delta D = \mid \Delta D_{max+} \mid + \mid \Delta D_{max-} \mid . \qquad (2)$$

### 3.2.1.3  Volumetric Performance
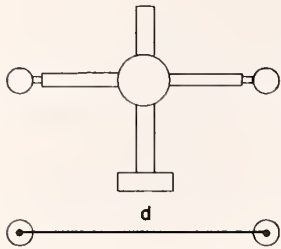


**Figure 11. B89.1.12M test for volumetric performance.**

The third B89.1.12M test, volumetric performance, deals with the ability of the machine to properly measure relative distance (indirectly intercomparing the length scales on the various machine axes and testing for imperfections of machine geometry).

In this test, a fixed-value, uncalibrated ball bar with a length 100 mm shorter than the machine axis is used.  At least four points are probed on each ball to locate its center and the distance between the center of the two balls is calculated.  For CMM's with nearly cubic workzones, the process is repeated at each of twenty orientations of the ball bar located at edges, face diagonals and body diagonals of the cube.  The "tolerance" on "volumetric performance" is the absolute value of the range of observed values for the 20 orientations.

Thus this "volumetric performance" test gives an estimate of the maximum distance error anywhere within the arbitrary-metric space of the machine:

$$\Delta d = \mid d_{max} - d_{min} \mid . \qquad (3)$$

### 3.2.1.4  Bi-Directional Length Measurement Capability
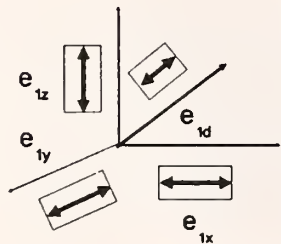


**Figure 12. B89.1.12M test for bi-directional length accuracy.**

The fourth B89.1.12M machine performance test, bi-directional length measurement capability, deals with the ability of the machine to properly measure extension in rectilinear modes.

In this test, a single gage block of calibrated length of the order of 25 mm is located where parts are typically measured within the workzone.  The length of the gage block is measured at each of four orientations of the block, three parallel to the axes of the machine and one parallel to an off-axis diagonal.  The tolerance on bi-directional length measurement capability is the maximum deviation of the observed values from the known value of the length of the gage block.

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This "bi-directional length measurement capability" performance test thus gives an estimate of the maximum error in such a measurement of extension of rectilinear-type objects:

$$\Delta e_x = | \, e_{obs} - e_{true} \, |_{max} .$$

(4)

### 3.2.1.5 Point-to-Point Probing Performance

The fifth B89.1.12M machine performance test, point-to-point probing performance, length measurement capability, deals with the ability of the machine to properly measure locations in radial directions. According to the standard, the test is devised to establish the magnitude of errors contributed by probing sequences for probes used in the point-to-point mode.



**Figure 13. B89.1.12M test for point-to-point probing.**
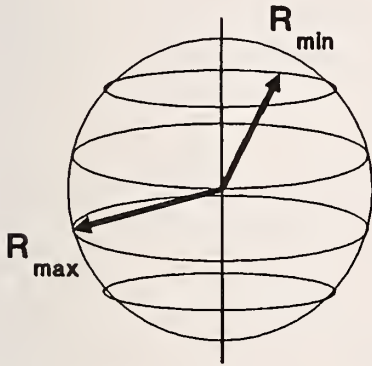
In this test, a roundness-calibrated precision reference ball is located within the workzone. The ball is probed at each of 49 points, 12 equally spaced on each of four circles of specified elevations plus one at the upper pole. From the set of 49 points, a sphere center is calculated and a radius determined for each measurement point.

The point-to-point performance is the maximum radius minus the minimum radius. This is functionally equivalent to the range of deviations of the 49 radii from the actual, but unknown, radius. Thus this "point-to-point probing" performance test gives an estimate of the maximum error in a measurement of the radial extension of an object in any orientation:

$$\Delta e_R = | \, e_{Rmax} - e_{Rmin} \, | .$$

(5)

### 3.2.2 Relation of B89.1.12M Performance Tests to Dimensions of Parts Measured

Each of the five B89.1.12M performance tests just described assesses the ability of a coordinate measuring machine to accurately perform one of the four fundamental types of dimensional measurements, displacement, position, distance, and extension, three of which correspond to dimensions of physical objects, such as the features of an automobile engine block shown schematically here.

■ The "measuring repeatability" performance test assesses the ability of the CMM to repeatably measure position of features such as that of a cylinder hole relative to the part
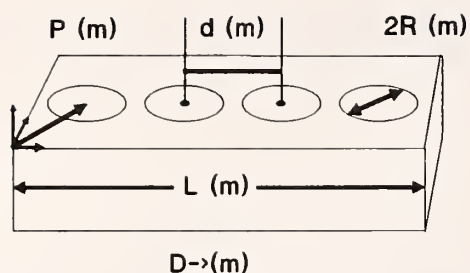
P (m)   d (m)   2R (m)

L (m)

D→(m)

**Figure 14. Example features of an automobile engine block by length dimensional type.**

coordinate system of the block (shown here as a corner and in practice defined by features such as dowel holes and pan-plate surfaces).

■ The "volumetric performance" test assesses the ability of the CMM to uniformly measure relative distance such as that between the centers of successive cylinder holes independent of their orientation.

■ The "bi-directional length measurement accuracy" performance test assesses the ability of the CMM to accurately measure extension of features such as the length of the block. (This is the "rectilinear" type of extension where L(left) and R(right) simply denote any pair of opposite-facing plane parallel boundaries).

■ The "point-to-point probing" performance test assesses the ability of the CMM to accurately measure radial extension type features such as the internal diameter of cylinder holes, the external diameter of the mating pistons, as well as the locations of planes to centers such as feature F in figure 8. (This is the "radial" type of extension where L(left) and C(center) simply denote any location on the surface of a sphere relative to the center).

■ Finally, the "linear displacement accuracy" performance test assesses the ability of the CMM to accurately measure displacement itself, relative to the international standard of length, as well as displacement-, position-, distance-, and extension-type features on objects.

## 3.3  Intercomparison of Standards for CMM Performance Testing

The principal standards dealing with performance evaluation of coordinate measuring machines are those of Germany (VDI/VDE 2617), Britain (BS 6808), and Japan (JIS B7440) as well as that of the United States (ASME B89.1.12) just discussed. Table 11 summarizes the test artifacts each of these standards specifies and the measurement quantity to which each artifact-based test corresponds.

For assessment of what it terms "length measuring uncertainty," the German VDI/VDE 2617 standard allows a step gauge or gauge blocks used in either one of two modes: bi-directional (here called the high-option, corresponding to what is here called an extension-type mode); or uni-directional (here called the low-option, corresponding to what is here called a displacement-type mode).

For assessment of what it terms "length measurement uncertainty," the British BS 6808 standard allows either a step gage or gauge blocks required to be used in a bi-directional

(extension-type) mode; the standard explicitly forbids use of the uni-directional (displacement) mode for assessing length measurement uncertainty.

**Table 11.** Comparison of the Principal Standards for CMM Performance Evaluation in Terms of the Artifacts They Specify and the Type of Length Measurement to Which Each Artifact-Based Test Corresponds

| Artifact Specified | Length Mode | VDI/VDE 2617 | BS 6808 | JIS B7440 | ASME B89.1.12 |
|---|---|---|---|---|---|
| Step Gage Gage Block | Extension [1] (L-to-R) | High Option | Required | Required | Required |
| Step Gage Gage Block | Displacement (L-to-L) | Low Option | Forbidden | Required | Option 2 |
| Ball Bar | Distance (C-to-C) | ... | ... | ... | Required |
| Laser Interferometer | Displacement (L-to-L) | Reqd for Axes > 1 m | ... | ... | Option 1 |
| Reference Ball | Position (C-to-O) | Required | ... | ... | Required |
| Reference Ball | Extension [2] (L-to-C) | Required | ... | ... | Required |

[1]  This is the "rectilinear" type of extension where L(left) and R(right) simply denote any pair of opposite-facing plane parallel boundaries.

[2]  This is the "radial" type of extension where L(left) and C(center) simply denote any location on the surface of a sphere relative to the center.

For assessment of what it terms "length measuring accuracy," the Japanese JIS B7440 standard allows either a step gauge or gauge blocks required to be used in a bi-directional (extension) mode. In addition, for assessment of "linear displacement accuracy," the Japanese standard allows either a step gage or "block gauges" used in a uni-directional (displacement) mode.

For assessment of what it terms "bi-directional length measurement accuracy," the American ASME B89.1.12(1990) standard requires a gauge block used in a bi-directional (extension) mode. In addition, the American standard specifies a number of other tests. For assessment of "linear displacement accuracy," the American standard allows use of either a laser interferometer (a displacement-measuring device) or a step gauge used in a uni-directional (displacement) mode. For assessment of "volumetric performance," this standard suggests use of a ball bar, a device which provides a fixed but uncalibrated distance. For assessment of "position repeatability," it requires use of an uncalibrated precision reference ball, which provides for a well-defined location (i.e., position). And, finally, for assessment of "point-to-point probing," it requires use of a roundness-calibrated precision reference ball (which allows a measure of non- or omni-directional extension). In general, the test of CMM performance required by the majority of these standards deals with the evaluation of the capability to measure extension; only the U.S. standard specifies tests for all four unit-of-length-based quantities.

As indicated by the table, while described by slightly different nomenclature (the British standard calling it "length measurement uncertainty," the Japanese "length measuring accuracy," and the American "bi-directional length measuring accuracy"), three of these four national standards specify either a step gauge, gauge blocks, or end standards used in a bi-directional mode for measurement of the extension.

The German standard does not require, but allows as an option, bi-directional measurements, for assessing "length measuring capability". Since that standard defines "length" as only "the distance between parallel planes", independent of whether they are the same- or opposite-facing, bi-directional (extension) and uni-directional (displacement) measurements are each allowed.

In addition to tests of extension-measurement capabilities, the Japanese, U.S., and German standards have functional equivalents of the U.S.'s separate "linear displacement accuracy" test, the U.S. standard explicitly allowing the optional use for the test of either laser interferometer systems or artifacts such as step gauges used in a uni-directional mode.

Only the U.S. standard specifies separate tests related to each of the unit-of-length-based measurement quantities which a CMM has the capability to measure, including distance and position as well as extension and displacement. For assessment of what it calls "volumetric performance," the U.S. standard requires use of gage blocks or a ball bar, a device which provides a fixed but uncalibrated distance. For assessment of "position repeatability," it requires use of a precision reference ball, which provides for a well-defined location relative to an origin or reference datum (i.e., position). And, finally, for assessment of "point-to-point probing," it requires use of a precision reference ball, which — while its radius is not calibrated — allows one to set an upper limit on omni-directional or radial extension errors, in contrast to the bi-directional or longitudinal extension described above.

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As shown in the appendix, section 3.6, however, the bi-directional step gage referred to in other national standards can be used for measurements in each of the four length modes. In either case, however, that of application of the U.S. standard with its variety of reference artifacts (end standards, ball bars, and precision reference spheres) or use of a bi-directional step gage as outlined in the appendix, one may begin to separate sources of error by type as a basis for remedial efforts or improved machine-system design.

## 3.4  Conclusion

With modern computer-controlled coordinate measuring machines and the parts they inspect being widely traded internationally, evaluation of the performance of CMM's has become an important matter for their producers and users. As a result, standards bodies in four major industrial nations — the United States, Japan, Germany, and Great Britain — have independently developed documentary standards for the evaluation of the performance of CMM's.

This chapter intercompares the tests prescribed by each of these standards in terms of a previously reported taxonomy of length-based measurements which: 1) makes distinctions among extension, distance, position, and displacement types of length measurement; and 2) shows that these types are ordered such that — in CMM measurements — errors propagate from the simplest type, displacement, through position and distance, to the most complex type, extension, which is the type most often thought of as "length."

The principal result of this intercomparison is to show that while the principal measure of accuracy in CMM's is a test of ability to measure an extension type of feature (with 3 of the standards requiring such a bi-directional test and the fourth allowing a uni-directional displacement-type test as an acceptable alternative), only the U.S. standard requires tests for each of the four length-measurement types. In doing so, it has in common with the other standards the most important measure of CMM accuracy while also providing a more comprehensive and unambiguous basis for a separation-of-variables analysis of overall system performance.

## 3.5 Appendix: Relations of Errors Specific to Types of Length Measurements

This appendix outlines briefly how each of the various types of length-based dimensional quantities is susceptible to type-specific errors which, in certain cases, propagate and compound in succession as described in detail in reference [1]. The example here is that of a coordinate measuring machine equipped with laser interferometers as scales and a touch-fire probe used to measure an extension-type feature such as the length of a block.

The first and most basic measurement this CMM must be able to make is that of displacement, where the machine accurately measures the change in location of its own moving element relative to its internal scale, in this case, a laser interferometer system. The errors in this measurement are shown formally as:

$$\Delta D = \Delta D\ (\delta D, \delta\lambda), \tag{A1}$$

which says that the total error in the displacement measurement $\Delta D$ is a function of $\delta\lambda$, the error in the vacuum wavelength of the laser, plus $\delta D$, errors specific to the displacement measurement, such as improper dead-path compensation and variations in the index of refraction along the path of the moving mirror.

The second basic measurement this CMM must be able to make is that of position, where the machine accurately measures the location of a feature, that is the center of that feature, relative to an origin, that is, a reference datum, fixed relative to the frame of the machine. The errors in this measurement are shown formally as:

$$\Delta P = \Delta P\ (\delta P,\ \Delta D), \tag{A2}$$

which says that the total error in the position measurement $\Delta P$ is a function of $\Delta D$, the total error in displacement measurement, plus $\delta P$, errors specific to the position measurement, such as Abbe error due to carriage tilt and non-directional variation in probing.

The third basic measurement this CMM must be able to make is that of distance, where the machine accurately measures the separation of the centers of two features on an object, whether artifact standard or manufactured part. The errors in this measurement are shown formally as:

$$\Delta d = \Delta d\ (\delta d,\ \Delta P), \tag{A3}$$

which says that the total error in the distance measurement $\Delta d$ is a function of $\Delta P$, the total error in position measurements, plus $\delta d$, errors specific to the distance measurement, such as the cosine error of misalignment between the axis of the measuring machine and that of the object.

Finally, the fourth and last basic measurement this CMM must be able to make is that of extension, where the machine accurately measures the separation of two opposite-facing boundaries of a feature, such as that of the end faces of a block. The errors in this measurement are shown formally as:

$$\Delta e = \Delta e \ (\delta d, \Delta e), \tag{A4}$$

which says that the total error in the extension measurement $\Delta e$ is a function of $\Delta d$, the total error in distance measurement, plus $\delta e$, which are errors specific to the extension measurement, such as those due to improper compensation for the finite extent of the probe.

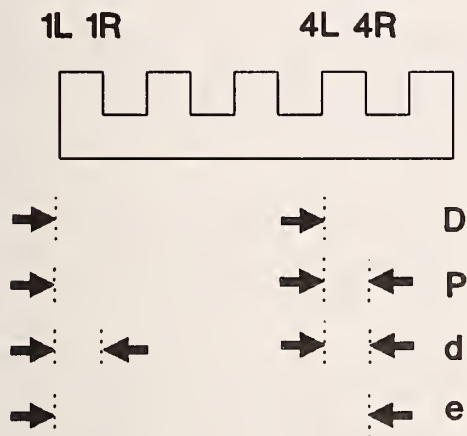## 3.6 Appendix: Use of a Step Gauge in Four Measurement-Quantity Modes



**Figure 15.** Use of step gage in displacement, position, distance, and extension modes of length measurement.

This appendix shows how, depending on the number and relative orientations of the boundaries of features involved: 1) which the four pure types of length-based measurements various features of step gages most closely correspond; and 2) how a bi-directional step gauge may be used to perform performance tests corresponding to these types.

First, a feature of the gage which has as its boundaries, for example, the left face of the first step (1L) and the left face of another step (e.g., 4L) corresponds to the blind-hole D in figure 8 and is here called a displacement-type feature.

This use of the term displacement is in the same sense that the U.S. B89 standard uses measurement of such features to assess what it calls "linear displacement accuracy," with a uni-directional approach to feature boundaries. (It should be noted, however, that measurements of displacement-type features can be susceptible to errors associated with those of position per se, such as errors due to Abbe offset combined with stage pitch).

Second, a feature of the gage which has as its boundaries, for example, the left face of the first step when that face is treated as a reference datum or origin of coordinates (1L) to the center of another step ( ½[4R+4L] ) is here called a radial-extension type feature. This feature corresponds to determination of the separation of a boundary from a center (and vice versa), and corresponds to feature F in figure 8. Errors associated with measurement of such features are evaluated by means of the B89 "point-to-point" performance test.

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Third, a feature of the gage which has as its boundaries the center of the first step ( [ ½[1L + 1R] ) and the center of another step ( e.g., ½[4L+4R] ) is what is here called a distance-type feature, since it involves the separation of the centers of two features. This corresponds to feature C in figure 8. Note, however, that when one of the features is formally designated an origin of part coordinates, the location of the second center relative to the first is then a position-type feature.

Finally, a feature which has as its boundaries a face of one step (1L) and an opposite face of another step (e.g., 4R), corresponding to feature A in figure 8, is an extension-type feature.

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# 5 Appendix: NIST Policy on Statements of Uncertainty Associated With Measurement Results[1]

A measurement result is complete only when accompanied by a quantitative statement of its uncertainty. This policy requires that NIST measurement results be accompanied by such statements and that a uniform approach to expressing measurement uncertainty be followed.

## 1. Background

Since the early 1980's, an international consensus has been developing on a uniform approach to the expression of uncertainty in measurement. Many of NIST's sister national standards laboratories as well as a number of important metrological organizations, including the Western European Calibration Cooperation (WECC) and EUROMET, have adopted the approach recommended by the International Committee for Weights and Measures (CIPM) in 1981 [1] and reaffirmed by the CIPM in 1986 [2].

Equally important, the CIPM approach has come into use in a significant number of areas at NIST and is also becoming accepted in U.S. industry. For example, the National Conference of Standards Laboratories (NCSL) is using it to develop a Recommended Practice on measurement uncertainty for NCSL member laboratories.

The CIPM approach is based on Recommendation INC-1 (1980) of the Working Group on the Statement of Uncertainties [3]. This group was convened in 1980 by the International Bureau of Weights and Measures (BIPM) in response to a request by the CIPM. More recently, at the request of the CIPM, a joint BIPM/IEC/ISO/OIML working group developed a comprehensive reference document on the general application of the CIPM approach titled *Guide to the Expression of Uncertainty in Measurement* [4] (IEC: International Electrotechnical Commission; ISO: International Organization for Standardization; OIML: International Organization of Legal Metrology). The development of the *Guide* is providing further impetus to the worldwide adoption of the CIPM approach.

## 2. Policy

All NIST measurement results are to be accompanied by quantitative statements of uncertainty. To ensure that such statements are consistent with each other and with present international practice, this NIST policy adopts in substance the approach to expressing measurement uncertainty recommended by the International Committee for Weights and Measures (CIPM). The CIPM approach as adapted for use by NIST is:

1) *Standard Uncertainty*: Represent each component of uncertainty that contributes to the uncertainty of the measurement result by an estimated standard deviation $u_i$, termed **standard uncertainty**, equal to the positive square root of the estimated variance $u_i^2$.

2) *Combined Standard Uncertainty*: Determine the **combined standard uncertainty** $u_c$ of the measurement result, taken to represent the estimated standard deviation of the result, by combining the individual standard uncertainties $u_i$ (and covariances as appropriate) using the usual "root-sum-of-squares" method, or equivalent established and documented methods.

   Commonly, $u_c$ is used for reporting results of determinations of fundamental constants, fundamental metrological research, and international comparisons of realizations of SI units.

3) *Expanded Uncertainty*: Determine an **expanded uncertainty** $U$ by multiplying $u_c$ by a **coverage factor** $k$: $U = ku_c$. The purpose of $U$ is to provide an interval $y - U$ to $y + U$ about the result $y$ within which the value of $Y$, the specific quantity subject to measurement and estimated by $y$, can be asserted to lie with a high level of confidence. Thus one can confidently assert that $y - U \leq Y \leq y + U$, or equivalently, $Y = y \pm U$.

Use expanded uncertainty $U$ to report the results of all NIST measurements other than those for which $u_c$ has traditionally been employed. To be consistent with current international practice, the value of $k$ to be used at NIST for calculating $U$ is, by convention, $k = 2$. Values of $k$ other than two are only to be used for specific applications dictated by established and documented requirements.

4) *Reporting Uncertainty*: Report $U$ together with the coverage factor $k$ used to obtain it, or report $u_c$.

When reporting a measurement result and its uncertainty, include the following information in the report itself or by referring to a published document:

- A list of all components of standard uncertainty, together with their degrees of freedom where appropriate, and the resulting value of $u_c$. The components should be identified according to the method used to estimate their numerical values:

    A.    those which are evaluated by statistical methods,

    B.    those which are evaluated by other means.

- A detailed description of how each component of standard uncertainty was evaluated.

- A description of how $k$ was chosen when $k$ is not taken equal to two.

It is often desirable to provide a probability interpretation, such as a level of confidence, for the interval defined by $U$ or $u_c$. When this is done, the basis for such a statement must be given.

Additional guidance on the use of the CIPM approach at NIST may be found in *Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results* [5]. A more detailed discussion of the CIPM approach is given in the *Guide to the Expression of Uncertainty in Measurement* [4]. Classic expositions of the statistical evaluation of measurement processes are given in references [6-8].

## 3. Responsibilities

a. Operating Unit Directors are responsible for compliance with this policy.

b. The Statistical Engineering Division, Computing and Applied Mathematics Laboratory, is responsible for providing technical advice on statistical methods for evaluating and expressing the uncertainty of NIST measurement results.

c. NIST Editorial Review Boards are responsible for ensuring that statements of measurement uncertainty are included in NIST publications and other technical outputs under their jurisdiction which report measurement results and that such statements are in conformity with this policy.

d. The Calibrations Advisory Group is responsible for ensuring that calibration and test reports and other technical outputs under its jurisdiction are in compliance with this policy.

e. The Standard Reference Materials and Standard Reference Data programs are responsible for ensuring that technical outputs under their jurisdiction are in compliance with this policy.

f. Authors, as part of the process of preparing manuscripts and other technical outputs, are responsible for formulating measurement uncertainty statements consistent with this policy. These statements must be present in drafts submitted for NIST review and approval.

## 4. Exceptions

It is understood that any valid statistical method that is technically justified under the existing circumstances may be used to determine the equivalent of $u_i$, $u_c$, or $U$. Further, it is recognized that international, national, or contractual agreements to which NIST is a party may occasionally require deviation from this policy. In both cases, the report of uncertainty must document what was done and why.

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[5]  B. N. Taylor and C. E. Kuyatt, *Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results*, NIST Technical Note 1297, prepared under the auspices of the NIST Ad Hoc Committee on Uncertainty Statements (U.S. Government Printing Office, Washington, DC, 1992).

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